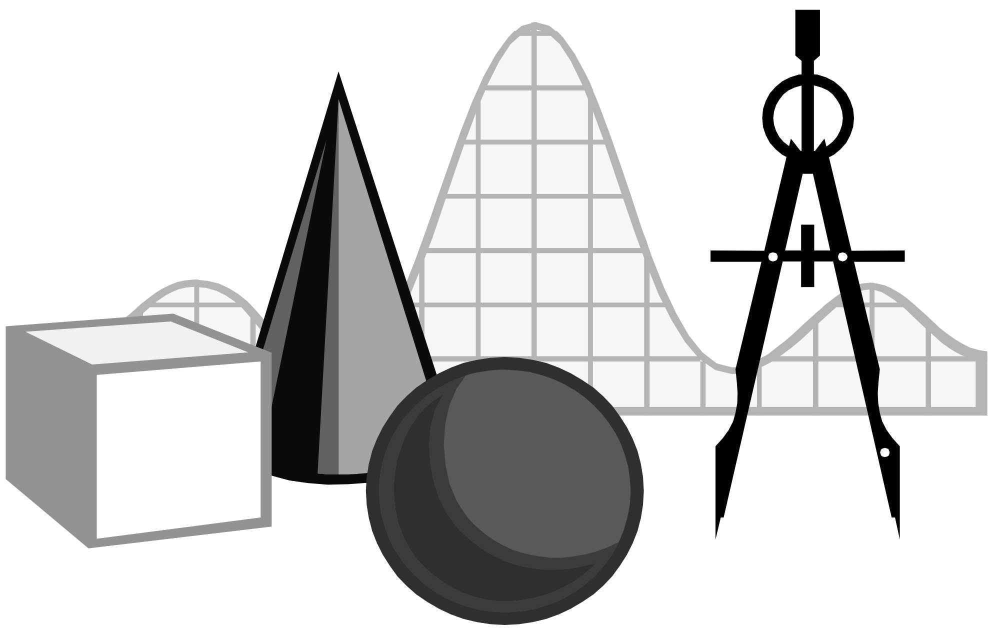
MATHEMATICS STANDARDS OF LEARNING ENHANCED SCOPE AND SEQUENCE

Grade 8



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Introduction

The *Mathematics Standards of Learning Enhanced Scope and Sequence* is a resource intended to help teachers align their classroom instruction with the Mathematics Standards of Learning that were adopted by the Board of Education in October 2001. The Mathematics Enhanced Scope and Sequence is organized by topics from the original Scope and Sequence document and includes the content of the Standards of Learning and the essential knowledge and skills from the Curriculum Framework. In addition, the Enhanced Scope and Sequence provides teachers with sample lesson plans that are aligned with the essential knowledge and skills in the Curriculum Framework.

School divisions and teachers can use the Enhanced Scope and Sequence as a resource for developing sound curricular and instructional programs. These materials are intended as examples of how the knowledge and skills might be presented to students in a sequence of lessons that has been aligned with the Standards of Learning. Teachers who use the Enhanced Scope and Sequence should correlate the essential knowledge and skills with available instructional resources as noted in the materials and determine the pacing of instruction as appropriate. This resource is not a complete curriculum and is neither required nor prescriptive, but it can be a valuable instructional tool.

The Enhanced Scope and Sequence contains the following:

- Units organized by topics from the original Mathematics Scope and Sequence
- Essential knowledge and skills from the Mathematics Standards of Learning Curriculum Framework
- Related Standards of Learning
- Sample lesson plans containing
    - Instructional activities
    - Sample assessments
    - Follow-up/extensions
    - Related resources
    - Related released SOL test items.

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Organizing Topic Number Sense/Number Theory

Standards of Learning

8.1 The student will
- a) simplify numerical expressions involving positive exponents, using rational numbers, order of operations, and properties of operations with real numbers;
- b) recognize, square represent, compare, and order numbers expressed in scientific notation; and
- c) compare and order decimals, fractions, percents, and numbers written in scientific notation.

8.2 The student will describe orally and in writing the relationship between the subsets of the real number system.

8.3 The student will solve practical problems involving rational numbers, percents, ratios, and proportions. Problems will be of varying complexities and will involve real-life data, such as finding a discount and discount prices and balancing a checkbook.

8.4 The student will apply the order of operations to evaluate algebraic expressions for given replacement values of the variables. Problems will be limited to positive exponents.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Simplify numerical expressions containing exponents where the base is a rational number and the exponent is a positive whole number, using the order of operations and properties of operations with real numbers.	____________________
• Recognize, represent, compare, and order rational numbers expressed in scientific notation, using both positive and negative numbers.	____________________
• Compare and order fractions, decimals, percents, and numbers written in scientific notation.	____________________
• Describe orally and in writing the relationships among sets of Natural or Counting Numbers, Whole Numbers, Integers, Rational Numbers, Irrational Numbers, and Real Numbers.	____________________
• Illustrate the relationships among the subsets of the real number system by using graphic organizers such as Venn diagrams.	____________________
• Identify the subsets of the real number system to which a given number belongs.	____________________
• Determine whether a given number is a member of a particular subset of the real number system, and explain why.	____________________
• Describe each subset of the set of real numbers.	____________________

- Solve practical problems by using computation procedures for whole numbers, integers, rational numbers, percents, ratios, and proportions. ____________________
- Maintain a checkbook and check registry for five or fewer transactions. ____________________
- Compute a discount and the resulting (sale) price for one discount. ____________________

Ordering Numbers

Reporting category Number and Number Sense
Overview Students arrange numbers from least to greatest.
Related Standard of Learning 8.1

Objective

- The student will compare and order fractions, decimals, percents, and numbers written in scientific notation.

Materials needed

- Sheets of paper with different numbers in scientific notation written on them, one copy for each student

Instructional activity

1. Distribute a sheet of paper with a different number in scientific notation to each student.
2. Have the students convert the number to a fraction, decimal, and percent.
3. Have the students arrange themselves in a line in numerical order, using the number on their papers. Discuss any corrections that need to be made, if necessary.
4. Repeat the activity with different numbers beginning as fractions, then decimals, and finally, percents. Have the students convert them to scientific notation before arranging themselves in order.

Sample assessment

- Discuss the different ways to compare numbers as the students are putting them in order.

Follow-up/extension

- Have the students add the numbers they were given, put the answer in scientific notation, and then arrange the sums in order.

Organizing Numbers

Reporting category Number and Number Sense
Overview Students organize numbers into subsets of the real number system.
Related Standard of Learning 8.2

Objective

- The student will use a Venn diagram to illustrate the relationships among the subsets of the real number system.

Materials needed

- "Real Numbers," one copy for each pair of students
- Scissors
- "Subsets of the Real Number System," one copy for each pair of students
- "Venn Diagram of the Real Number System," one copy for each pair of students
- Glue or tape

Instructional activity

1. Arrange the students in pairs. Give a copy of "Real Numbers" to each pair, and have them cut apart the numbers.
2. Have the students sort the numbers in different unspecified sets. Circulate among the groups and ask them to explain the process they used to sort the numbers.
3. Have a class discussion on the attributes of the sets of numbers.
4. Hand out a copy of "Subsets of the Real Number System" to each group. Have the students cut out the subsets and arrange them in any order.
5. Have the students sort the numbers into the different subsets. Discussion should take place here on numbers that could belong in more than one subset.
6. Have a class discussion on the properties of each subset. Then, have the students sort the numbers as rational or irrational.
7. Have the students arrange the rational numbers into rational numbers, integers, whole numbers, and/or natural numbers. This can be done by arranging the names of the subsets as shown on the right. The numbers can then be placed on one or more of the subsets.
8. Give a copy of "Venn Diagram of the Real Number System" to each group.
9. Have the students place the names of the subsets in the appropriate boxes in the Venn diagram. Then, have them place the numbers in the appropriate subsets.
10. Circulate among the students. When a group has the diagram completed correctly, have them glue or tape the names and numbers onto the paper.
11. Have the students add one or two numbers in writing to each subset of the real number system on their diagram.

Rational Numbers	**Integers**
Whole Numbers	**Natural Numbers**

Sample assessment

- Circulate among students as they are arranging their numbers into the subsets. Assess the completed diagrams for each group. Have the students write a summary of the relationship among the subsets of the real number system.

Follow-up/extension

- Have the students create their own graphic organizer to illustrate the relationships among the subsets of the real number system.

Real Numbers

0	$\frac{1}{2}$	**0.7**	**1**
–3	$\sqrt{2}$	**–0.9**	π
–4.267	$-\frac{5}{17}$	**14.8**	**–8**

Subsets of the Real Number System

Rational Numbers

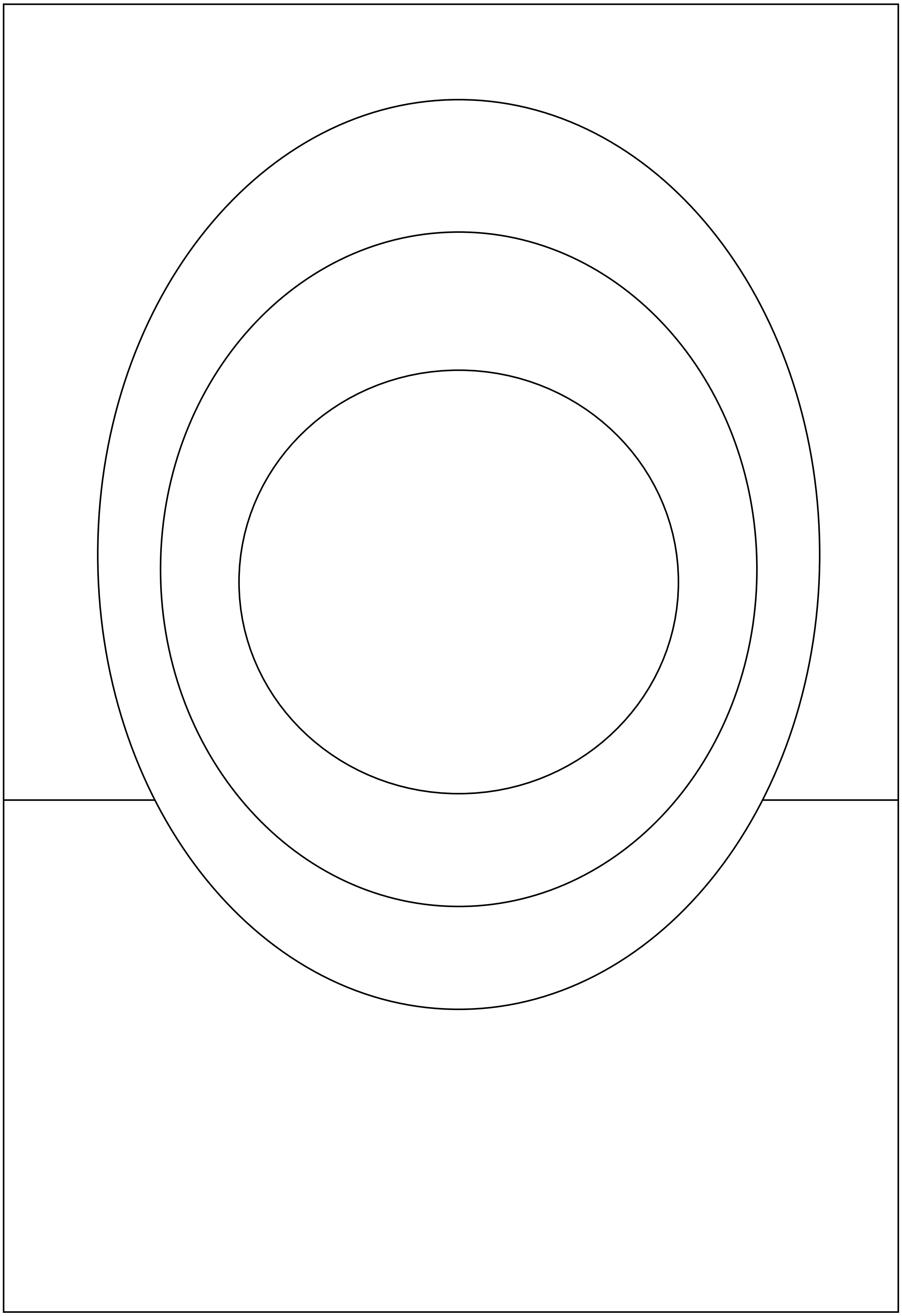
Irrational Numbers

Integers

Whole Numbers

Natural Numbers

Venn Diagram of the Real Number System



Playground Problem

Reporting category Computation and Estimation

Overview Students apply what they have learned about adding and subtracting fractions to a real-life situation.

Related Standard of Learning 8.3

Objective

- The student will estimate and develop strategies for adding and subtracting fractions.
- The student will gain experience in selecting the appropriate operation to solve problems.

Materials needed

- Individual sets of fraction strips or other fraction manipulatives
- "Playground Problem, Current Playground Map," one copy for each group
- "Future Playground Map," one copyfor each group
- Transparencies of "Playground Problem" and "Current Playground Map"
- Chart paper
- Markers
- Tape

Instructional activity

1. Introduce the problem by showing and explaining the transparency "Current Playground Map." Then show the transparency "Playground Problem," and review the problem with the students so they will understand what they need to do.
2. Discuss with the class named fractional areas of the playground. Be sure students understand the meaning of "two equal parcels (sections) of land."
3. Divide the class into four groups for a roundtable. Have the students in each group take turns counting the fractional parts of an area of the current playground and marking it with a fraction. The map should be passed from person to person until every playground area is marked with the appropriate fraction.
4. Have the students work in pairs to solve the problem. At each table, have the pairs discuss their solutions and come to a consensus.
5. Give each group the handout "Future Playground Map" and some markers. Have them draw/record their solution and show how they found it, justifying their answer. Monitor the groups as they work.
6. Tape the "Future Playground Maps" to the wall, and allow the groups to circulate to examine the solutions of the other groups. Allow them to write suggestions or comments on the sheets as they make their examinations.
7. Have the groups return to their tables and discuss with the whole class their observations and comments. Be sure to discuss any discrepancies.

Sample assessment

- As students work, circulate among the groups, observe the work, and listen to the discussions. Be sure everyone understands the task. Check solutions for completeness as well as accuracy. Be sure each group records how they found the solution or justified their answer.

Playground Problem

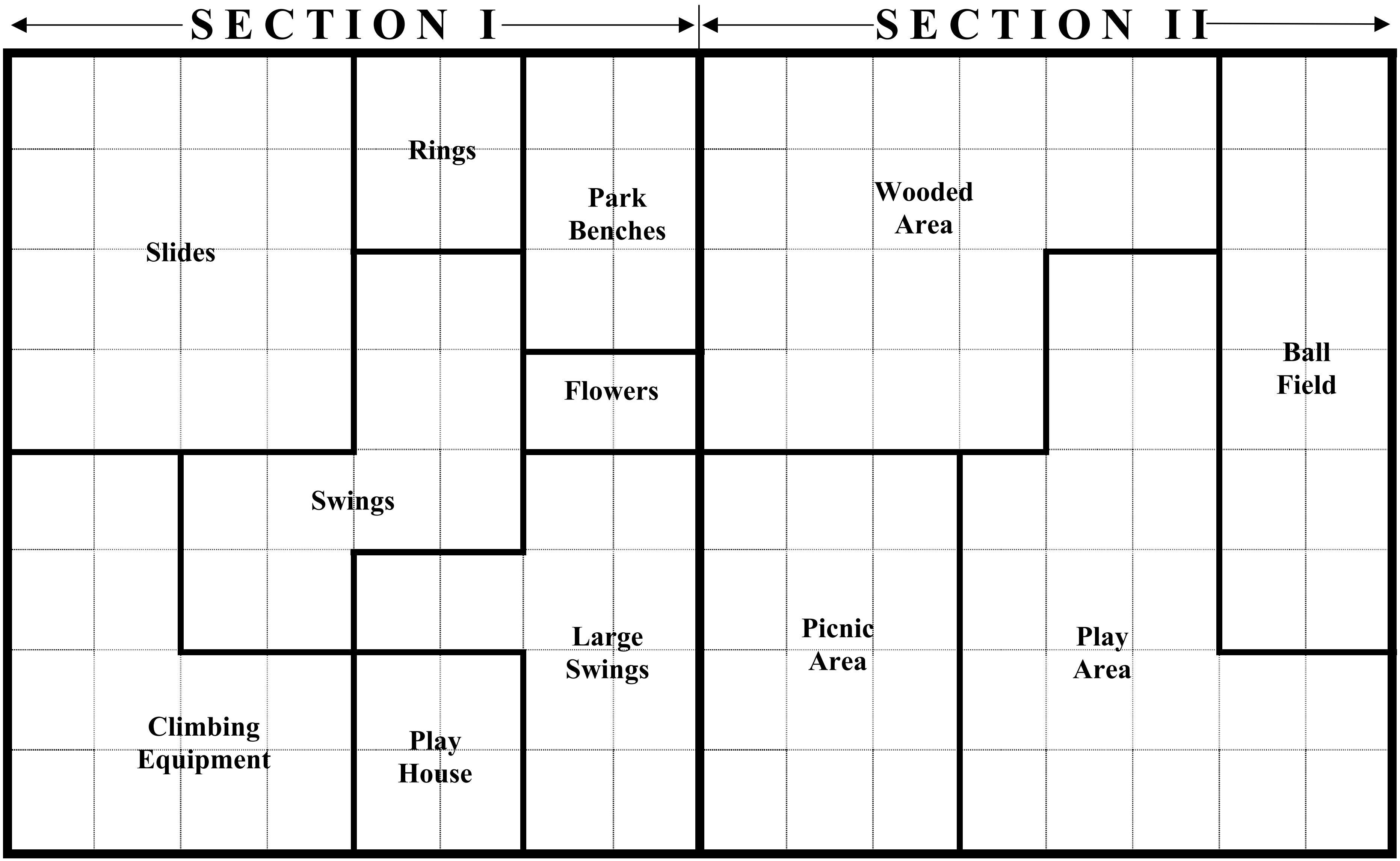
The playground at Central Elementary School is formed from two equal parcels (sections) of land, which have been subdivided into 12 assorted areas with specific purposes, as shown on the map. The P.T.A. has agreed to donate money to redesign and update the playground. They want the existing 12 areas combined into four larger areas with the following specifications:

1. After the renovation, all 12 of the existing areas of the playground will be eliminated, and the whole playground will be divided into four new areas.
2. The ball field area will be joined to one other area, and together they will make up $\frac{1}{2}$ of one of the two equal parcels or sections of land. This will be the new "Play Area."
3. The area containing the Play House will be combined with two other areas to form the new "Primary Playground." This area will comprise $\frac{13}{32}$ of one of the two sections.
4. The area that holds the slides will be increased to equal $\frac{1}{2}$ of a section. This will be the new "Playground Equipment Area."
5. The rest of the land in that section will be added to the park benches area. This will be the new "Park & Picnic Area."
6. You will be able to walk through each of the four new areas of the playground without having to cross another area.

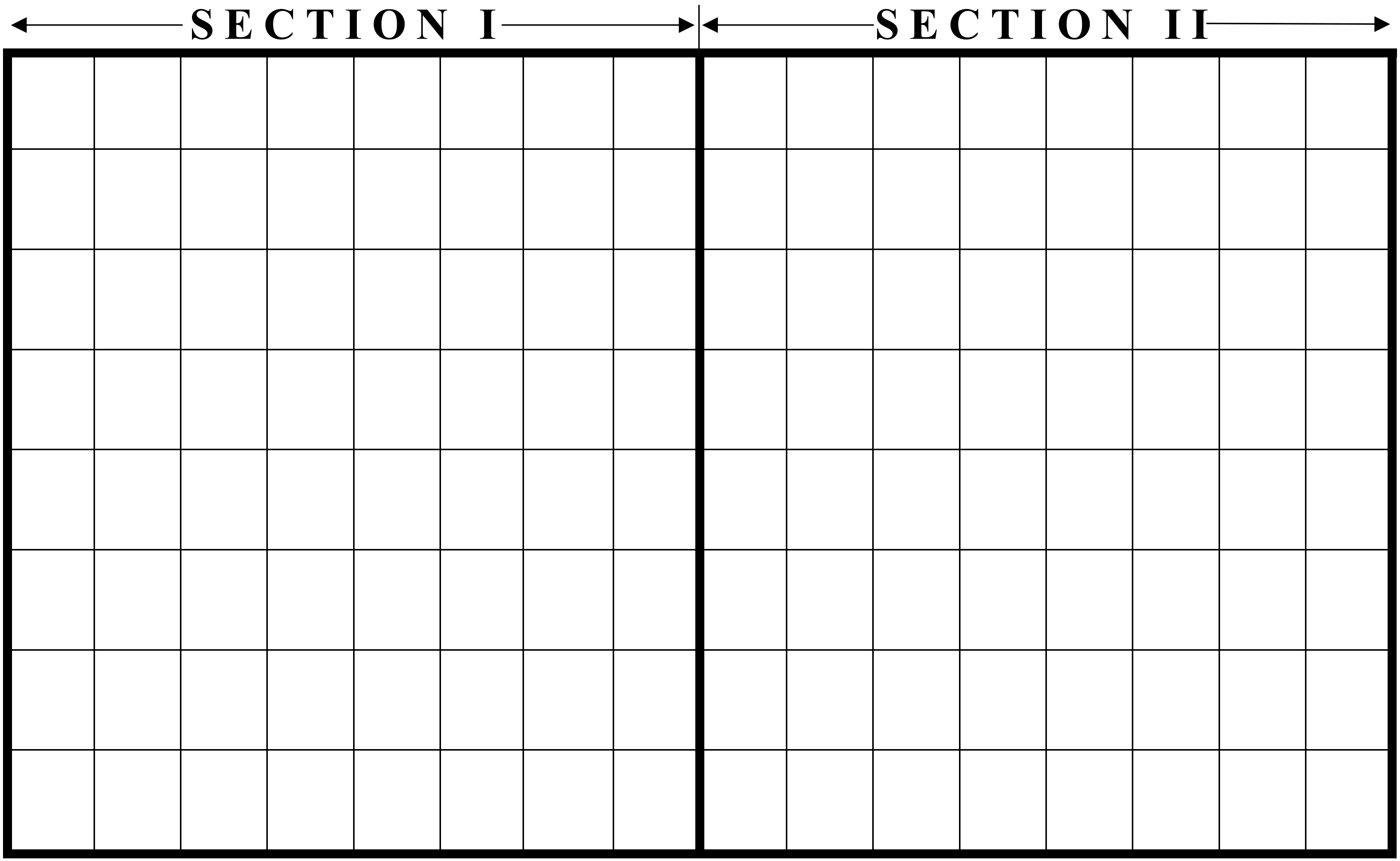
Use the clues above to find out which areas of the original playground were combined to make the four new areas. Explain your answer.

7. Draw a map of the new playground, and outline each of the four new areas. What fraction of the total playground area will be in each of the four new areas?
8. Do you agree with the design of the playground? Explain any changes you would make in the location of equipment or park areas.

Current Playground Map



Future Playground Map



Measuring Cups

Reporting category Computation and Estimation

Overview Students determine how many fractional parts of a cup are contained in a specified cup quantity.

Related Standard of Learning 8.3

Objectives

- The student will identify the whole and the unit in a given division problem.
- The student will solve expressions involving division of fractions.
- The student will discuss patterns that develop when dividing fractions.
- The student will discuss why the procedure of "invert and multiply" is used.

Materials needed

- A full set of measuring cups for each pair or small group of students
- Food coloring (if using clear measuring cups)

Instructional activity

1. *Initiating Activity:* Ask students to develop a word problem for the expression $10 \div 5$. It should take about a minute for them to do this task.
2. Ask a few students to share their word problem with the whole class. Hold a short discussion about the language used in these word problems. Common phrases that may arise are: "Ten items are shared fairly with 5 people." or "How many 5s are in 10?"
3. Give the students the expression $10 \div 5$. Ask, What does 10 refer to? What does 5 refer to? When dividing, we always refer to the whole. Thus, in this expression, 10 refers to the whole, and 5 refers to the size of the measuring unit or set.
4. Continue this discussion by giving the students the expression $\frac{3}{4} \div \frac{1}{2}$. Ask, What does $\frac{3}{4}$ refer to? What does $\frac{1}{2}$ refer to? In this expression, $\frac{3}{4}$ refers to the whole, and $\frac{1}{2}$ refers to the size of the measuring unit or set.
5. Ask students to use the previous example of $10 \div 5$ to determine how they would solve $\frac{3}{4} \div \frac{1}{2}$. It may be necessary to help students make the connection and to tell them that they need to determine how many $\frac{1}{2}$s are in $\frac{3}{4}$.
6. *Demonstration:* Fill a $\frac{3}{4}$ measuring cup full of water. (Use food coloring in the water if clear measuring cups are available.) Ask students, "How many $\frac{1}{2}$ cups are in this $\frac{3}{4}$ cup of water?" Another way of asking this is, "How many times can you pour the $\frac{3}{4}$ cup of water into the $\frac{1}{2}$ cup?"

7. After several students have given and defended their answer, physically pour water from the $\frac{3}{4}$ cup into the $\frac{1}{2}$ cup. Once the $\frac{1}{2}$ cup is full, ask students: How many $\frac{1}{2}$ cups are in the $\frac{3}{4}$ cup? They should recognize that there is *more than* one $\frac{1}{2}$ cup in a $\frac{3}{4}$ cup because there is water remaining in the $\frac{3}{4}$ cup.
8. Pose the question, "What should we do with the remaining water?" At this point, someone may make the common mistake of saying that because $\frac{1}{4}$ of a cup of water remains, there are one-and-one-fourth $\frac{1}{2}$s in $\frac{3}{4}$. Students should physically pour the remaining water into the $\frac{1}{2}$ cup. They should then recognize that the remaining water ($\frac{1}{4}$ of a cup) fills *half* of the $\frac{1}{2}$ cup.
9. Repeat the beginning question: "How many $\frac{1}{2}$ cups are in $\frac{3}{4}$ of a cup?" (Solution: There are one and one-half $\frac{1}{2}$ cups in $\frac{3}{4}$ of a cup.) Then, repeat the original question: "How many $\frac{1}{2}$s are in $\frac{3}{4}$?" (Solution: There are one and one-half $\frac{1}{2}$s in $\frac{3}{4}$.)
10. Have students select a partner and solve the following expressions. Allow them to use measuring cups or fraction bars, if available. Partners should discuss the method used to solve each expression.
    a. $\frac{1}{2} \div \frac{1}{3}$ (How many $\frac{1}{3}$ cups are in $\frac{1}{2}$ of a cup?)
    b. $\frac{1}{3} \div \frac{1}{4}$ (How many $\frac{1}{4}$ cups are in $\frac{1}{3}$ of a cup?)
    c. $1 \div \frac{2}{3}$ (How many $\frac{2}{3}$ cups are in 1 cup?)
    d. $2 \div \frac{3}{4}$ (How many $\frac{3}{4}$ cups are in 2 cups?)
    e. $\frac{3}{4} \div \frac{1}{3}$ (How many $\frac{1}{3}$ cups are in $\frac{3}{4}$ of a cup?)
    f. $1\frac{1}{3} \div \frac{1}{2}$ (How many $\frac{1}{2}$ cups are in $1\frac{1}{3}$ cups?)
    g. $2\frac{1}{4} \div \frac{1}{3}$ (How many $\frac{1}{3}$ cups are in $2\frac{1}{4}$ cups?)
    h. $1\frac{1}{2} \div \frac{1}{3}$ (How many $\frac{1}{3}$ cups are in $1\frac{1}{2}$ cups?)
    i. $1\frac{3}{4} \div \frac{3}{4}$ (How many $\frac{3}{4}$ cups are in $1\frac{3}{4}$ cups?)
    j. $1\frac{2}{3} \div \frac{1}{4}$ (How many $\frac{1}{4}$ cups are in $1\frac{2}{3}$ cups?)
11. As a class, verify the results of each expression.

12. Have the students continue to work with their partners to solve the following expressions. Discuss the patterns that develop. As a class, verify the results of each expression.
    a. $1 \div \frac{2}{5}$
    b. $2 \div \frac{2}{5}$
    c. $3 \div \frac{2}{5}$
    d. $4 \div \frac{2}{5}$

Note: Proportional reasoning indicates that there will be twice as many two-fifths in the second expression than in the first. Similarly, there will be three times as many two-fifths in the third expression than in the first. Also, students may notice that you can multiply the dividend by the denominator of the divisor and then divide this solution by the numerator of the divisor to arrive at the correct solution. Be certain to explain that the reason this method works is because division is the inverse of multiplication. Therefore, when we divide, it is the same as multiplying by the reciprocal. This notion helps to explain why we "invert and multiply."

Sample assessment

- While the pairs are working, circulate and listen to the discussions taking place. Watch how students are selecting a strategy or approaching the problem. If anyone is simply falling back on the "invert and multiply" algorithm, insist that they show you they understand the problem and its solution with concrete materials or in narrative form.

Follow-up/extension

- Write about how you would solve the following problem: A recipe calls for $1\frac{3}{4}$ cups of butter. Each stick of butter is $\frac{1}{2}$ of a cup. How many sticks of butter will be needed for this recipe?

Dividing Fractions, Using Pattern Blocks

Reporting category Computation and Estimation

Overview Students use pattern blocks to represent the whole and then determine a fractional amount of the whole.

Related Standard of Learning 8.3

Objective

- The student will be able to divide a whole number by a fraction.

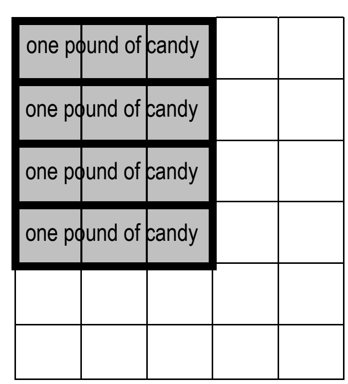
Materials needed

- Pattern blocks
- Graph/grid paper
- Colored pencils (optional)

Note: For easier management, put each pattern block set in a plastic storage bag for each student.

Instructional activity

1. *Initiating Activity:* Discuss with the class the questions: "What is division of whole numbers? What does 6 divided by 2 mean? What does 12 divided by 2 mean? What does 12 divided by 6 mean?" In asking these questions, you are trying to encourage student understanding that division describes how many of a given divisor there are in a given dividend. Have students represent (sketch) 6 divided by 3 and make a story problem for 6 divided by 3. Have volunteers read their story problems.
2. Have students work in groups to model, using pattern blocks, the solution to the following problem: "Duncan has four pounds of candy and decides to use it all to make $\frac{1}{3}$-pound bags to give away. How many bags can he make with his four pounds of candy?"
    a. Students may solve this problem by using four hexagons to model four pounds of candy. Using a rhombus to represent $\frac{1}{3}$ of a pound of candy, they will see that there are three rhombi in a hexagon and, therefore, twelve rhombi in four hexagons. Hence, one can make twelve $\frac{1}{3}$-pound bags of candy out of four pounds of candy.
    b. Another way students may solve this problem is to use graph paper to represent the problem. Since the problem uses division by thirds, discuss with the students why three equal-sized parts should be used to represent each pound of candy. Colored pencils may be used, if desired. Students should also write the problem as in the model at right.



$$4 \div \frac{1}{3} =$$

$$4 \bullet 3 = 12$$

3. Have small groups of students use pattern blocks to model the following problem and then represent it on graph paper: "Susan had three blocks of candy. She wants to divide each block into

$\frac{1}{6}$-size pieces of candy. How many pieces of candy will she be able to make?" Students may solve this problem by using three hexagonal blocks to represent the three blocks of candy. They may then use a triangular block to represent $\frac{1}{6}$ of a block and find the solution. (18 pieces of candy).

4. Have small groups of students use pattern blocks to model the following problem and then represent it on graph paper: "The Virginia Housing Company wants to divide five acres of land into $\frac{1}{2}$-acre lots. How many lots will there be?" Students may solve this problem by using five hexagonal blocks to represent the five acres of land. They may then use a trapezoidal block to represent a $\frac{1}{2}$-acre lot and find the solution. (10 lots).
5. Have students write a problem for 4 divided by $\frac{1}{6}$ and then solve, using pattern blocks and graph/grid paper.
6. Have students write a general rule for dividing a whole number by a unit fraction.

Note: If needed, the activity may be stopped here and briefly reviewed the next day before continuing.

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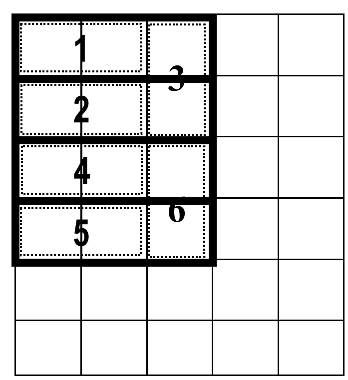
7. Have small groups of students use pattern blocks to model the following problem and then represent it on graph paper: "Mark has four packs of paper and wants to repackage them for his Boy Scout project into packs that are each $\frac{2}{3}$ the size of each original pack. How many new packs will he have?"
   a. Students may solve this problem by using four hexagonal blocks to represent the four original packs of paper. They may then use two rhombi to represent $\frac{2}{3}$ of a hexagonal block and find the solution. (six packs)
   b. Students may represent the problem on graph paper as in the model at right.



$$4 \div \frac{2}{3} = 6$$

8. Have small groups of students use pattern blocks to model the following problem and then represent it on graph paper: "For a science experiment, a class wants to cut six yards of yarn into $\frac{2}{3}$-yard pieces. How many pieces will they get?"
9. Have small groups of students write a problem for $8 \div \frac{2}{3}$, use pattern blocks to model it, and then represent it on graph paper to solve.
10. Have students write a general rule for dividing a whole number by $\frac{2}{3}$.

11. Have the students solve the following problems, using the procedures already established:

$$3 \div \frac{3}{4}$$

$$6 \div \frac{3}{4}$$

$$12 \div \frac{3}{4}$$

12. Have students write a general rule for dividing a whole number by $\frac{3}{4}$. Ask, Why do you have to divide by three? Why multiply by four?
13. Have students develop a rule for dividing any whole number by any fraction that is less than one.

Sample assessment

- During the activity, observe students as you walk around the room and check for understanding. At the end of the activity, students may respond in their math journals to the following prompt: "Describe a rule for dividing a whole number by a fraction. Describe common circumstances in which people divide by fractions."

Follow-up/extension

- Students should be encouraged to find examples of dividing by fractions in the real world. Another representation for pattern blocks is using available software. The following Web sites have "virtual" pattern blocks and activities:
  - http://www.arcytech.org/java/patterns/patterns_j.shtml
  - http://www.matti.usu.edu/nlvm/nav/frames_asid_169_g_1_t_2.html
  - http://step.k12.ca.us/community/fractions_institute/flash/pattern_block.html

Homework

- If this activity is used over two days, limit the first night's homework to problems involving the division of whole number by unit fractions. Answers should have whole number answers.

Ratio, Proportion, and Percent

(This lesson is derived from *Math Connects: Patterns, Functions, and Algebra)*

Reporting category	Computation and Estimation
Overview	Students will use ratios and proportions to solve problems.
Related Standards of Learning	8.3, 8.17

Objectives

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to
  - write problems that require establishing a relationship between ratios
  - solve problems by using proportions
  - solve practical problems by using computation procedures for whole numbers, integers, rational numbers, percents, ratios, and proportions.

Materials needed

- Protractor
- Metric ruler
- Graphing calculator

Instructional activity

Note: View the videotape from Math Connects: *Patterns, Functions, and Algebra.* Before class, have students complete the included worksheet. They will be measuring the side lengths and angles in right triangles.

1. Tell the students that their science project is to relate the rate of speed of a toy car traveling down an inclined plane to the steepness (slope) of the plane in order to investigate the potential hazards of highways built with varying grades of steepness. Ask, "When you see a sign on a mountain highway that said "Danger: 7% grade ahead," what does that mean?
2. Participants will begin to define *grade*, or *steepness*, or *slope,* using the ratio of one leg of a right triangle to another (vertical height to horizontal height). They will measure angles of triangles and side lengths to explore the concepts of ratio and proportions, percent, and similarity of triangles. The slope ratios will be defined as a function of the measure of one of the acute angles of the triangle. Points from the function will be plotted to determine a curve of best fit. It will then be revealed that this is a special function and will be displayed and traced on the graphing calculator.
3. Objectives include linear and angle measurement, ratio, proportion, percent, similarity, and reinforcement of the concept of function as displayed in a chart and graphically.
4. Have the students complete before class the included worksheet in which they will be measuring the side lengths and angles in right triangles.
5. During class, explore interesting graphs, using the completed worksheets and calculators.
6. Questions for reflection:
   - Cross multiplication is often used to solve proportions. Why is emphasizing cross multiplication risky?
   - What applications to real life can we use in helping students understand how important it is to know when and how to solve problems using ratios and/or proportions?

Math Connects:

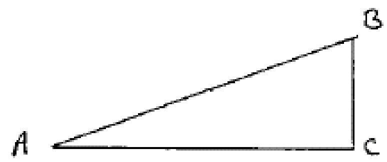
PATTERNS • FUNCTIONS • ALGEBRA

WORKSHEET FOR LESSON 5

Each triangle ABC has a right angle at C. For each triangle:

1. Measure angle A (M/A) in degrees
2. Measure AC and BC (side lengths) in millimeters.
3. Write the ratio $\frac{BC}{AC}$ as a fraction and as a decimal rounded to 2 decimal places

1)



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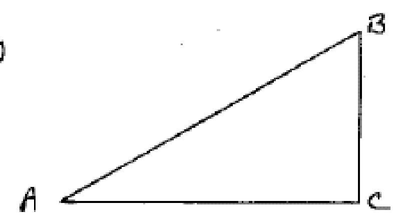

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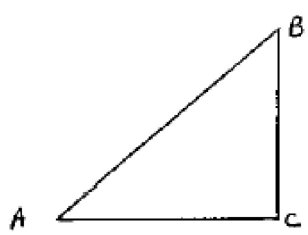

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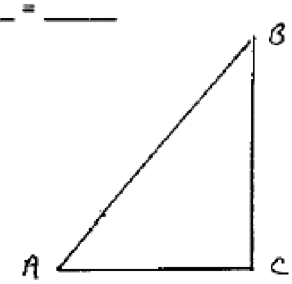

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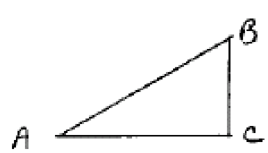

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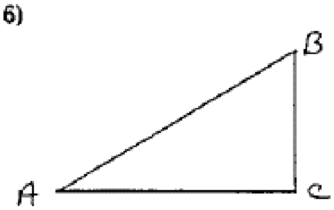

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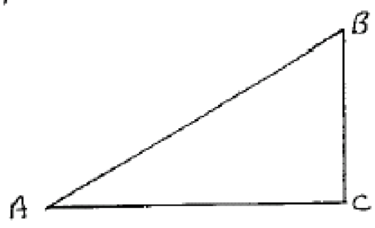
Math Connects:

PATTERNS • FUNCTIONS • ALGEBRA

7)



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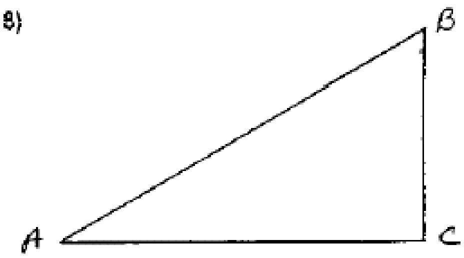

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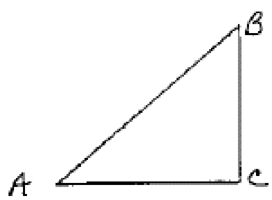

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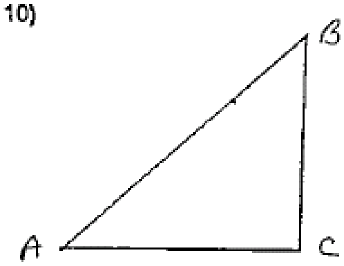

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BC = ________
BC/AC = ______ = ______

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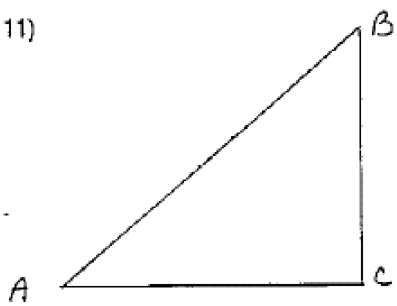

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BC = ________
BC/AC = ______ = ______

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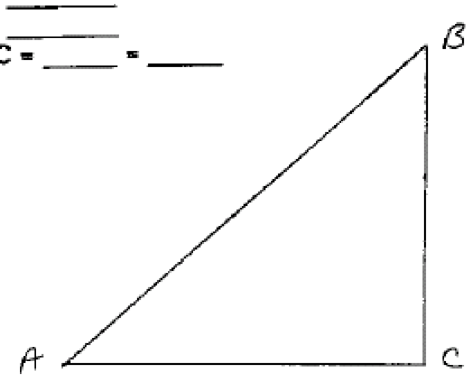

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Math Connects:
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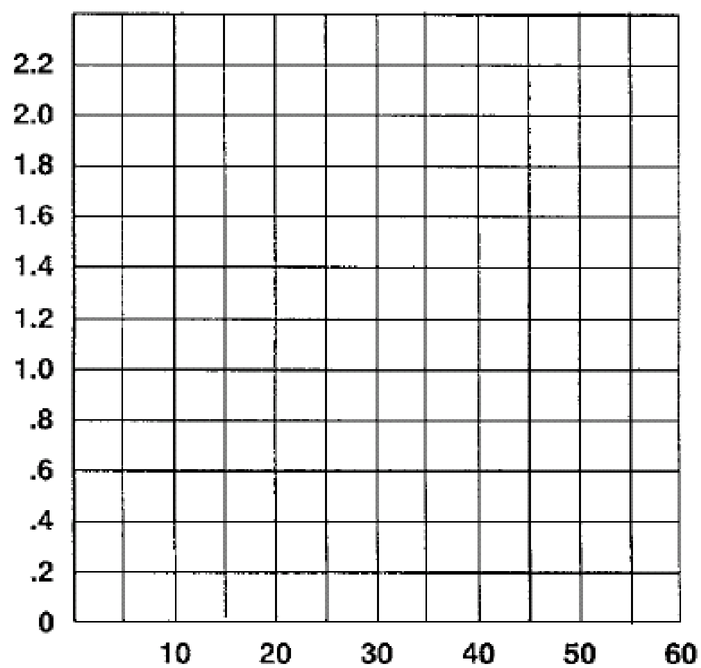
WORKSHEET FOR LESSON 5



Spending Money

Reporting category	Computation and Estimation
Overview	Students complete transactions in a checkbook registry for items purchased at a store.
Related Standard of Learning	8.3

Objectives
- The student will compute a discount, the resulting sale price, sales tax, and the resulting final price of an item.
- The student will maintain a checkbook registry.

Materials needed
- "Do You Like to Spend Money?" one copy for each student

Instructional activity
1. Hand out a copy of "Do You Like to Spend Money?" to each student and discuss the scenario and the task.
2. Review calculating percents of numbers with the students.
3. Have the students complete the discount table and checkbook registry alone or in pairs.
4. Have the students share their transactions, and determine which student ended with a balance close to $0.00.

Sample assessment
- Circulate among the students as they are completing their discount tables and checkbook registries. Discuss the different combinations of purchases they could make.

Follow-up/extension
- Ask the students if the task is possible to accomplish when only two items are purchased.

Do You Like to Spend Money?

Scenario: You are walking down the street and notice the following sign in the window of a store.

$50 will be yours if you can spend it all in this store!

You go into the store to get more details. A salesperson tells you that if you can purchase at least three different items, and the total sale is between $49.00 and $50.00, then you can have the items for free.

Task: Maintain a checkbook registry with a beginning balance of $50.00. A separate transaction must be made for each item you purchase. Use the discount table to calculate the total cost of each item. Choose items from the following display.

SALE!!! SALE!!! SALE!!! SALE!!! SALE!!!

	Original price	**Discount**
T-Shirts	**$12.00**	**25% off**
Shorts	**$15.00**	**20% off**
Jeans	**$35.00**	**40% off**
Sweatshirts	**$20.00**	**30% off**

Discount Table

Item	Original Price	Amount of Discount	Sale Price	Sales Tax (4.5%)	Final Price

Checkbook Registry

Transaction Description	Payment/Debit	Deposit/Credit	Balance

Patterns on Powers

(This lesson derived from *Math Connects: Patterns, Functions, and Algebra)*

Reporting category	Computation and Estimation
Overview	Students evaluate expressions.
Related Standard of Learning	8.4

Objective

- The student will substitute numbers for variables in an algebraic expression and simplify the expression by using the order of operations
- The student will apply the order of operations to evaluate formulas

Materials needed

- Graphing calculators
- "Patterns on Powers," one copy for each student
- Several sheets of paper to fold and tear

Instructional activity

1. Have the students watch the video from *Math Connects: Patterns, Functions, and Algebra.*
2. Distribute the handout, and have the students work through it.

Follow-up/extension

- Have the students answer the following question in their journals: "What further connection(s) can we make about 'patterns in powers'?" Give an example of something you might use to have students form conjectures about numbers raised to a power. (Hint: Look in *Patterns and Functions Addenda* series book.)

Patterns on Powers

1. What does "n^1/2" mean?

2. The meaning of raising a number to a negative power has been demonstrated in the video. Use a calculator to explore what it means to raise the following numbers to the 1/2 power:

4^1/2	18^1/2
9^1/2	25^1/2
10^1/2	33^1/2
16^1/2	81^1/2
100^1/2	121^1/2

3. What do you notice? What question do you ask yourself when finding the "square root" of a number?

4. Will the same process work for finding the cube root of a number? Explain your thinking.

5. 25 raised to what power equals 5?
   64 raised to what power equals 4?
   3 raised to what power equals 1/3?

6. What do you notice? What generalization can you make?

7. If your school board gave you the following choice of salary methods, which would you choose? Explain why this is your choice.
   Option A: One billion dollars per year plus health coverage, a yearly expense account of $1 million and a $100,000 computer (one time only).
   Option B: The amount of money obtained by putting $1 on one square of a chessboard, $2 on the next, $4 on the next, $8 on the next, and so on until all 64 squares are filled.

8. Solve this problem another way. (Idea: Use technology, if you haven't already done so.)

9. At any point in time, is one option more advantageous than the other?

The Language of Algebra: Order of Operations

(Lesson © 2000 by Math.com)

Reporting category	Computation and Estimation
Overview	Students evaluate expressions.
Related Standard of Learning	8.4

Objectives

- The student will substitute numbers for variables in an algebraic expression and simplify the expression by using the order of operations
- The student will apply the order of operations to evaluate formulas

Materials needed

- a computer lab with a computer for every student or a computer with a viewing system that may be seen by all the students in the class

Instructional activity

1. Have students access the Internet Web site at http://www.math.com/school/subject2/lessons/S2U1L2GL.html
2. Students should work through “First Glance,” “In Depth,” “Examples,” and “Workout.”

Sample assessment

- “Workout” from step 2 above.

Round Robin

Reporting category	Number and Number Sense/Computation and Estimation
Overview	Students simplify expressions by using the order of operations.
Related Standards of Learning	8.1, 8.4

Objective

- The student will create numerical and algebraic expressions, and use the order of operations to simplify them.

Instructional activity

1. Review the order of operations with the students.
2. Have the students arrange their desks in a large circle.
3. On a piece of paper, have each student create a numerical expression involving at least three different operations. It may be necessary to limit the value of the numbers used.
4. Once each student has written an expression, the papers should be passed one student to the right. Each student should then rewrite the expression under the original with the first operation performed.
5. After a set amount of time (30 to 45 seconds), have the students pass the papers again to the right. Each student should check the work of the previous student, confer with that student, if necessary, and then perform the next operation. Continue this until the expression is simplified.
6. If a student receives an expression that is simplified, have him/her create an algebraic expression on the back of the paper, with replacement values for the variables. Have them continue passing the papers until all of the expressions have been simplified.

Sample assessment

- Circulate as the students are simplifying the expressions. Listen to student discussions if they are conferring about a problem. Use the student-created expressions as a quiz.

Follow-up/extension

Have the students create their own pneumonic device for remembering the order of operations. They can illustrate their version of PEMDAS and present it to the class.

Formula Stations

Reporting category Computation and Estimation
Overview Students collect real-life data to use in evaluating formulas.
Related Standard of Learning

Objective

- The student will apply the order of operations to evaluate formulas.

Materials needed

- Ruler
- Models of rectangular prisms, cylinders, pyramids, and cones
- "Recording Sheet —Formula Stations" one copy for each student

Instructional activity

1. Set up stations around the classroom with different objects at each. Make sure to have enough objects so that each team is able to measure something at all times.
2. Pair up the students, and give each a ruler and a copy of "Recording Sheet – Formula Stations."
3. Have the students rotate around the stations, measuring the parts of each solid indicated on the recording sheet.
4. Once all stations have been visited, have the students return to their desks. The teams should use their data to evaluate the formulas for surface area of each solid.
5. On the board, make a chart so that each team can record their calculated surface areas.

Object	Team 1	Team 2	Team 3	Team 4	Team 5
Prism #1					
Prism #2					
Prism #3					
Cylinder #1					
Cylinder #2					
Cylinder #3					
Pyramid #1					
Pyramid #2					
Pyramid #3					
Cone #1					
Cone #2					
Cone #3					

6. Discuss the different calculations, and make any corrections, if necessary.

Sample assessment

- Monitor students as they measure the objects at different stations. Make sure that they are measuring properly. Assist the students with evaluating the formulas, if necessary.

Follow-up/extension

- This activity could be done using different measurements to evaluate the formulas for area, perimeter, and volume.

Recording Sheet – Formula Stations

Station 1	Length (l)	Width (w)	Height (h)	Surface area
Prism #1				
Prism #2				
Prism #3				

Station 2	Radius of base (r)	Height (h)	Surface area
Cylinder #1			
Cylinder #2			
Cylinder #3			

Station 3	Slant height (l)	Perimeter of base (p)	Area of base (B)	Surface Area
Pyramid #1				
Pyramid #2				
Pyramid #3				

Station 4	Slant height (l)	Radius of base (r)	Surface area
Cone #1			
Cone #2			
Cone #3			

Perfectly Squared

Reporting category	Computation and Estimation
Overview	Students identify perfect squares and estimate square roots.
Related Standard of Learning	8.5

Objectives

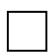
- The student will create models to identify the perfect squares from 0 to 100.
- The student will find the two consecutive whole numbers between which a square root lies.

Materials needed

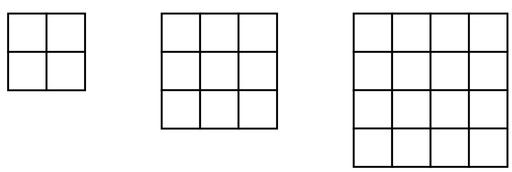
- grid paper (10 cm-by-10 cm)
- scissors

Instructional activity

1. Have the students cut the grid paper into 100 squares. (This can be done ahead to save time.)
2. Have the students use the fewest number of pieces of paper to make a square.



3. Have the students model the next three larger squares.



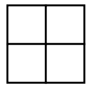
4. Discuss the dimensions of each square and the number of pieces used in each model. The definition of a *perfect square* can also be discussed at this time. For example, 9 is a perfect square because 9 pieces make a “perfect” 3-by-3 square, or $3 \times 3 = 9$.
5. Have the students continue modeling perfect squares until all are discovered up to 100. Record students’ answers on the board using the following table:

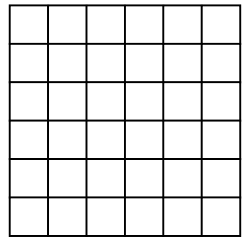
Dimensions of square	Number of square pieces used
1-by-1	1
2-by-2	4
3-by-3	9
4-by-4	16
5-by-5	25
.	.
.	.
.	.

6. Discuss the definition of a square root by using a model of a perfect square. For example, the square root of 25 can be found by making a square with 25 pieces. The length of a side of the square, 5, is the square root of 25.

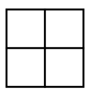
7. Have the students find the square roots of numbers using the square pieces. (Example: Find the square roots of 1, 4, and 36.)

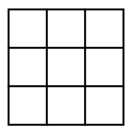
1 
1
$\sqrt{1} = 1$

2 
2
$\sqrt{4} = 2$

6 
6
$\sqrt{36} = 6$

8. Have the students try to find the square root of 6 using square pieces. Once they discover that it cannot be done, have them find the two perfect squares closest to 6.





Since $\sqrt{4} = 2$ and $\sqrt{9} = 3$, the $\sqrt{6}$ must be between 2 and 3.

9. Have the students find the two consecutive whole numbers between which the square roots of the given numbers lie: 7, 10, and 18.

$\sqrt{7}$ lies between 2 and 3.

$\sqrt{10}$ lies between 3 and 4.

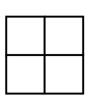
$\sqrt{18}$ lies between 4 and 5.

Sample assessment

- Circulate as students model the perfect squares. Ask for volunteers to draw pictures of their models on the board when finding the perfect squares up to 100.

Follow-up/extension

- To estimate square roots of numbers that are not perfect squares, model the following:
  - Find the two consecutive integers between which the $\sqrt{6}$ lies. Use square pieces to model the perfect squares of those integers.

2 
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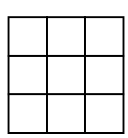
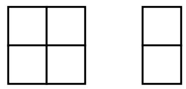
3 
3

- Only four pieces are needed to model the 2-by-2 square, and 9 pieces are needed to model the 3-by-3 square. When using six pieces to find $\sqrt{6}$, the 2-by-2 square can be made, but there will be two pieces left over.



- Since 5 more pieces are needed to make the 3-by-3 square, $\sqrt{6}$ is approximately $\frac{22}{5}$. The whole number, 2, comes from the size of the perfect square smaller than 6, and the $\frac{2}{5}$ is the ratio of the leftover square pieces to the number of square pieces needed to make the next perfect square larger than 6. Use a calculator to find $\sqrt{6} \approx 2.449\ldots$ and show the students how close the estimate is to the actual measure.

The Pythagorean Theorem

Reporting category Geometry
Overview Students verify and apply the Pythagorean Theorem.
Related Standard of Learning 8.10

Objectives

- The student will use square units to verify the Pythagorean Theorem.
- The student will find the measure of a missing side in a right triangle.
- The student will solve real-life problems involving right triangles.

Materials needed

- "The Pythagorean Theorem Model," on copy for each student
- Scissors
- "The Pythagorean Theorem Exercises," one copy for each student

Instructional activity

1. Give a copy of "The Pythagorean Theorem Model" worksheet to each student. Discuss the parts of the right triangle, including the hypotenuse and the legs.
2. Review the concept of a perfect square, and emphasize that the square units on each side of the triangle are the perfect squares of those sides.
3. Have each student cut out only the square units of the legs of the triangle. Once all of the squares have been cut apart, have the students place them on the square units of the hypotenuse. All of the squares will be filled. Discuss the variables in the Pythagorean Theorem at this time. (a and b are the measures of the legs, and c is the measure of the hypotenuse.)
4. Hand out a copy of "The Pythagorean Theorem Exercises" worksheet to each student. Do a couple of the problems with the students. Have the students complete the worksheet.

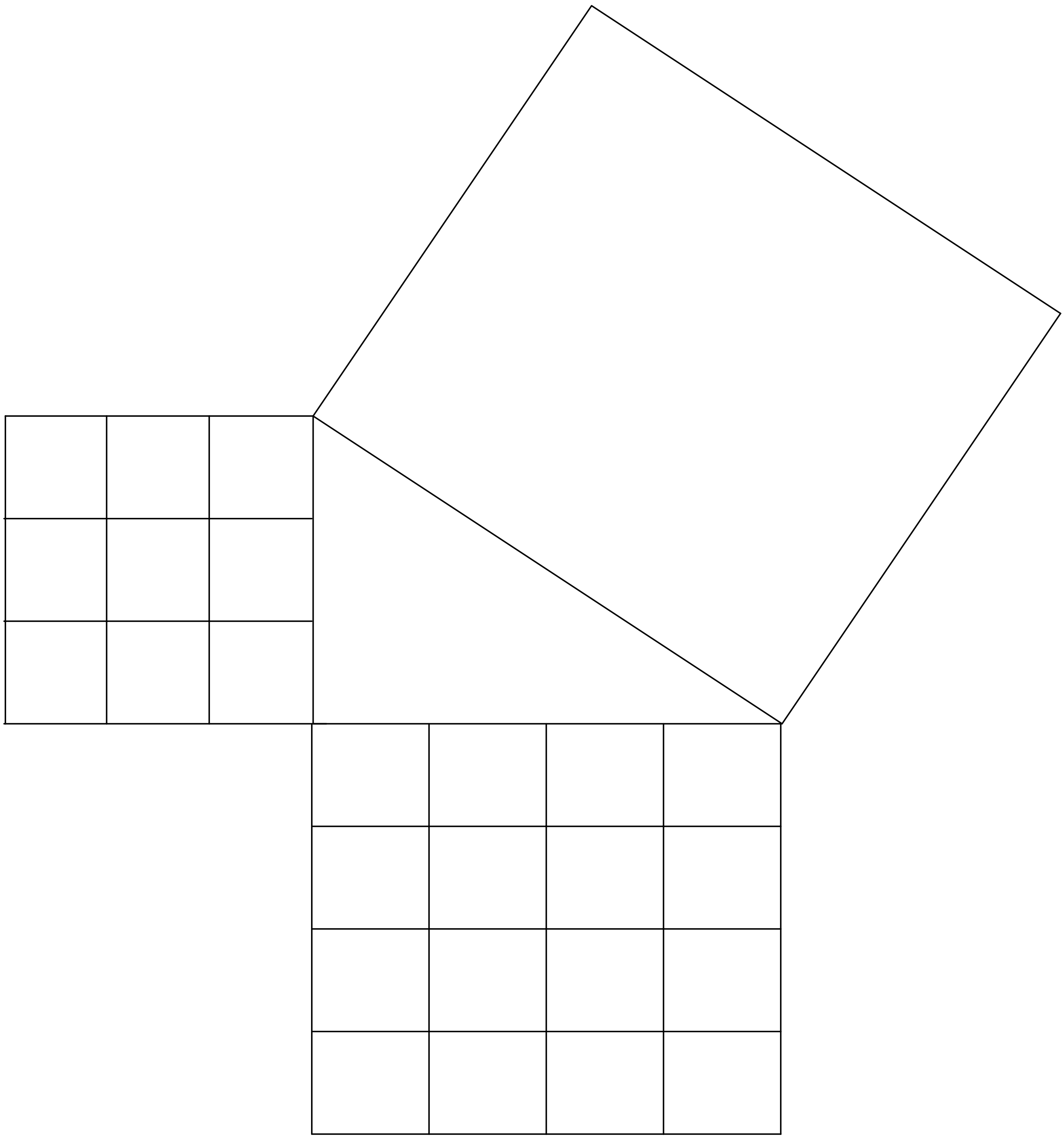
Sample assessment

- Circulate among the students as they are cutting the square units and rearranging them on the square of the hypotenuse. Check the answers to the exercises on the worksheet.

Follow-up/extension

- Have the students make up their own real-life problem that involves the Pythagorean Theorem to solve.

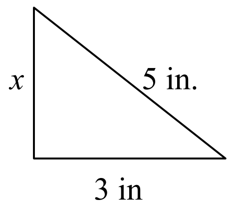
The Pythagorean Theorem Model



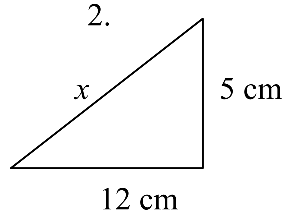
The Pythagorean Theorem Exercises

Find the length of the missing side in the following examples. Round answers to the nearest tenth, if necessary.

1.



2.



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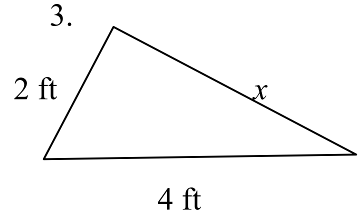

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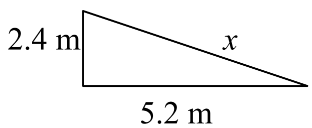


5. A 10-foot ladder is leaning against the side of a house. If the base of the ladder is 3 feet away from the house, how high up the side of the house will the ladder reach?

6. Rebecca left her house and walked 2 blocks east. She turned and walked 5 blocks north to get to the library. If each block is $\frac{1}{4}$ of a mile, how far is the direct route from Rebecca's house to the library?

Pythagoras of Samos

Reporting category Geometry

Overview Students engage in experiences that allow them to verify the Pythagorean Theorem and its converse. They are guided through several variations of proofs of the theorem.

Related Standard of Learning 8.10

Objectives

- The student will use and verify the Pythagorean Theorem.
- The student will find the measure of a missing side in a right triangle.
- The student will solve real-life problems involving right triangles.

Materials needed

- 11-pin geoboards or dot paper
- "Geoboard Exploration of Right Triangles," one copy for each student

Instructional activity

1. Review the definition of a right triangle with students. Review mathematical vocabulary associated with right triangles (hypotenuse, leg).
2. Put students into groups of 2 or 3.
3. On a transparent geoboard on the overhead projector, construct a right triangle in which one leg is horizontal and the other is vertical. Ask a participant to construct a square on each leg and then on the hypotenuse of the triangle. Ask participants to find the area of each square. It may be difficult for some students to recognize a way to find the area of the square on the hypotenuse.
4. Give students the handout entitled, "Geoboard Exploration of Right Triangles," and have students fill in the data as the teacher debriefs the examples with the whole class.
   - What patterns do you see?
   - Can you state the relationship in words? In symbols?
   - Do you think this is always true?
   - If you label the sides of the triangle, can you write a statement of what you think is true?
   - Does this procedure provide a proof that the relationship is always true?

Follow-up/extension

- **Egyptian Rope Stretching:** Students will use a rope with 13 knots tied at equal intervals as a simulated Egyptian artifact that applies the Pythagorean Theorem.
- Show students the rope with 13 knots tied at equal intervals. A picture of a rope very much like this one was noted in inscriptions in many of the tombs of ancient Egyptian pharaohs. Ask students what they think the purpose of a rope like this might have been.
- If students come up with the idea that the Egyptians used the rope to make a template for determining right angles, ask them to demonstrate. If they do not, ask two students to help the teacher demonstrate. Have one student hold knots #1 and #13 together. Have a second student hold

knot #8. All three students should stretch the rope and have the class observe the resulting shape. Allow students holding the same knots to make a different shape.

- Have students conjecture about how the Egyptians might have used a rope like this (to build right angles on the pyramids, to mark the boundaries of fields after the annual spring flooding of the Nile).
- Ask students what other numbers of knots in a rope might be used to serve the same purpose of forming a right triangle. For example, can one obtain the right triangle result with a rope that has 20 (19 spaces) equally spaced knots? Can one do it with a rope of 31 knots (30 spaces)?

Geoboard Exploration of Right Triangles

Length of side *a*	Length of side *b*	Length of hypotenuse, *c*	Area of square on leg *a*	Area of square on leg *b*	Area of square on hypotenuse *c*	$a^2 + b^2$

Sample released test items

Which of these numbers is a perfect square?

F 12
G 24
H 36
J 48

Between which two consecutive whole numbers will you find $\sqrt{82}$?

A 6 and 7
B 7 and 8
C 8 and 9
D 9 and 10

Between which two consecutive whole numbers does $\sqrt{42}$ lie?

A 5 and 6
B 6 and 7
C 7 and 8
D 8 and 9

Between what two consecutive whole numbers does $\sqrt{95}$ lie?

A 7 and 8
B 8 and 9
C 9 and 10
D 10 and 11

What is the value of $7a(a - 1) + 6$ when $a = 2$?

A 20
B 33
C 54
D 98

What is the value of $x^2(7 - x) + 2$ when $x = 5$?

F 52
G 100
H 152
J 172

What is the value of $n^2(m + r)$ if $m = 3$, $n = 2$, and $r = 4$?

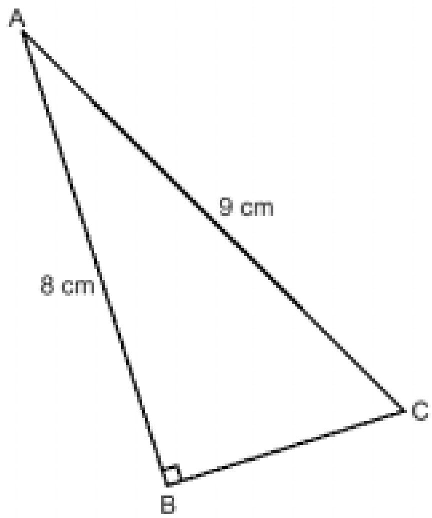
A 28
B 16
C 14
D 9

Which is equivalent to $(7 - 3)^3$?

A 316
B 64
C 12
D 4

What is the value of $3 + 7(2^3 - 6)^2$?

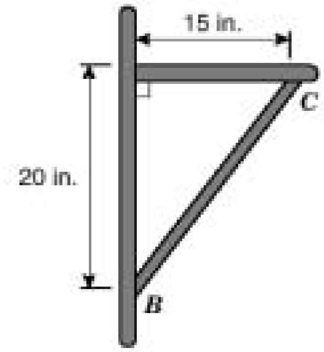
F 23
G 31
H 84
J 2,503



What is the length of $\overline{BC}$?

F 3 cm
G 7 cm
H $\sqrt{17}$ cm
J $\sqrt{115}$ cm

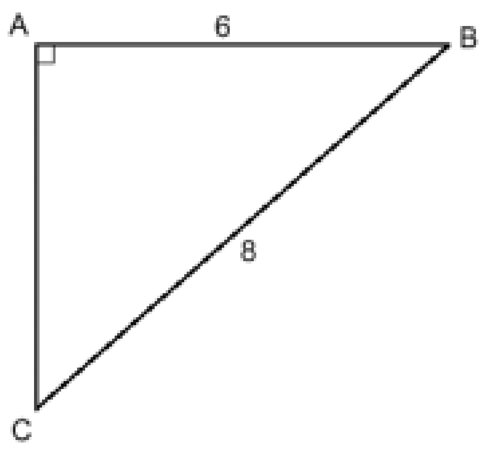
This is a cross section of the design of a bookshelf.



Which is closest to the length, in inches, of the brace indicated by $\overline{BC}$ in the sketch?

A 25 in.
B 30 in.
C 32.5 in.
D 35 in.

In Δ ABC, $\overline{AB}$ measures 6 centimeters and $\overline{BC}$ measures 8 centimeters.



What is the length of $\overline{AC}$?

F 1.41 cm
G 2 cm
H 5.29 cm
J 10 cm

Organizing Topic Measurement

Standards of Learning

8.6 The student will verify by measuring and describe the relationships among vertical angles, supplementary angles, and complementary angles and will measure and draw angles of less than 360°.

8.7 The student will investigate and solve practical problems involving volume and surface area of rectangular solids (prisms), cylinders, cones, and pyramids.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Measure angles of less than 360° to the nearest degree, using appropriate tools.	______________________
• Identify and describe the relationships among the angles formed by two intersecting lines.	______________________
• Identify and describe pairs of angles that are vertical.	______________________
• Identify and describe pairs of angles that are supplementary.	______________________
• Identify and describe pairs of angles that are complementary.	______________________

What Does a Name Measure Up To?

Reporting category	Measurement
Overview	Students measure angles formed by letters in their names
Related Standard of Learning	8.6

Objective:
- The student will measure angles of less than 360° to the nearest degree, using a protractor.

Materials needed
- Ruler
- Protractor (or other measuring device)

Instructional activity
1. Have the students write their name in capital letters, using only line segments. Make sure that they write the letters large enough to measure the angles formed by the line segments.
2. Have the students measure each angle formed in all of the letters. Have them label each angle with the correct measurement to the nearest degree.
3. Review acute, right, obtuse, and straight angles. Discuss the angles formed that were more than 180° and the different ways to calculate their measurements.

Sample assessment
- As the students are measuring their angles, make sure that they are using the measuring device properly. When measuring the angles more than 180°, lines can lightly be drawn separating the angle into a straight angle and one less than 180° so that a protractor can be used.

Follow-up/extension
- Challenge the students to draw their letters in their name in such a way that each letter contains an acute, right, obtuse, and straight angle.

What's Your Angle?

Reporting category Measurement

Overview Students measure angles formed by intersecting lines and investigate relationships between angle pairs.

Related Standard of Learning 8.6

Objectives

- The student will measure angles of less than 360º to the nearest degree, using appropriate tools
- The student will identify and describe the relationship among the angles formed by two intersecting lines
- The student will identify and describe pairs of angles that are vertical
- The student will identify and describe pairs of angles that are supplementary
- The student will identify and describe pairs of angles that are complementary.

Materials needed

- Protractor

Instructional activity

1. Review measuring angles with a protractor. As students measure angles, have them name the angles using several conventions. Ask students to identify the type of angle (acute, right, obtuse, reflex, straight).
2. Allow students to work on the activity sheet in pairs. Assess student understanding by circulating through the room and questioning student pairs. The definitions of the angles pairs may not be as elegant as the yours, but as long as the students' definition is mathematically correct and the student can justify it, acknowledge it as correct and acceptable.

Sample assessment

- Conduct student interviews to determine the level of understanding of each student pair.

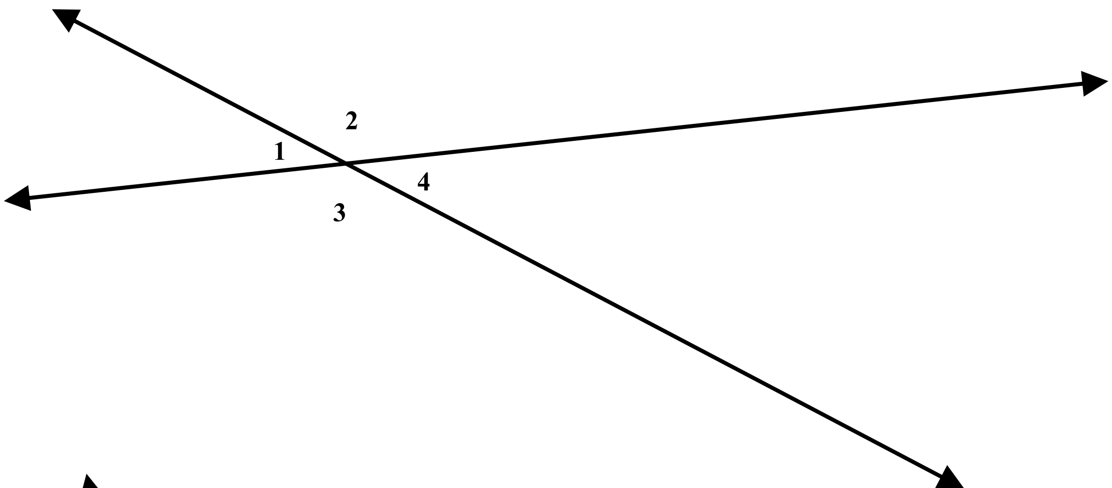
Follow-up/extension

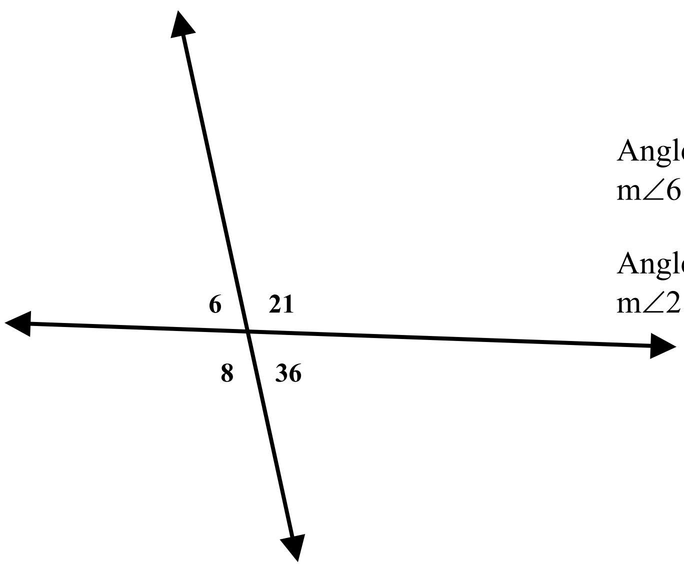
- Have each member of the student pair draw examples of vertical, supplementary, and complementary angles. Have the other student measure to verify (or show as a counterexample) whether or not the drawing is correct.

What's Your Angle?

∠1 and ∠4 are vertical angles. Measure them. m∠1 = _______ m∠4 = ________

∠2 and ∠3 are vertical angles. Measure them. m∠2 = _______ m<3 = ________





Angles 6 and 36 are vertical angles. Measure them.
m∠6 = _______ m∠36 = ________

Angles 21 and 8 are vertical angles. Measure them.
m∠21 = _______ m∠8 = ________

What seems to be the relationship between the measures of vertical angles? ___________________

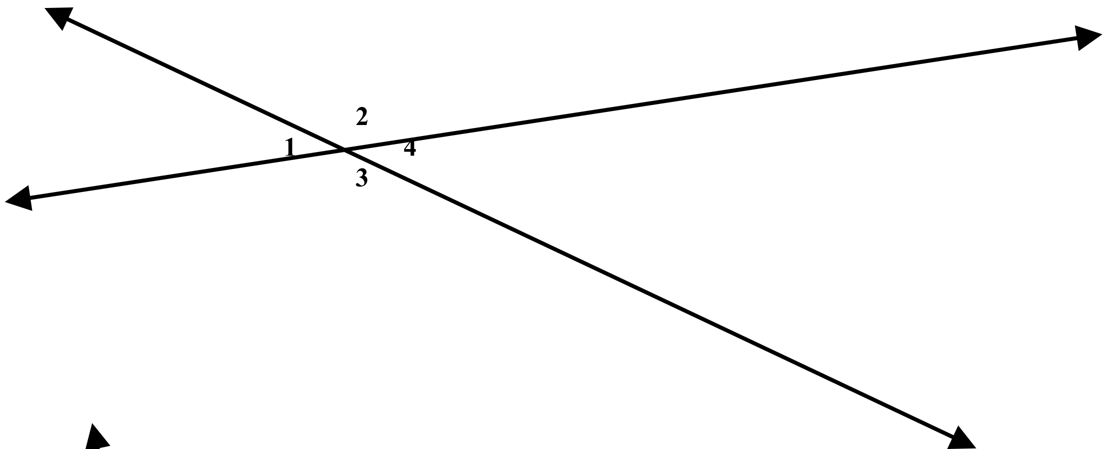
Can you identify vertical angles? Write a descriptive definition of vertical angles. Let your teacher read your description and comment on it.

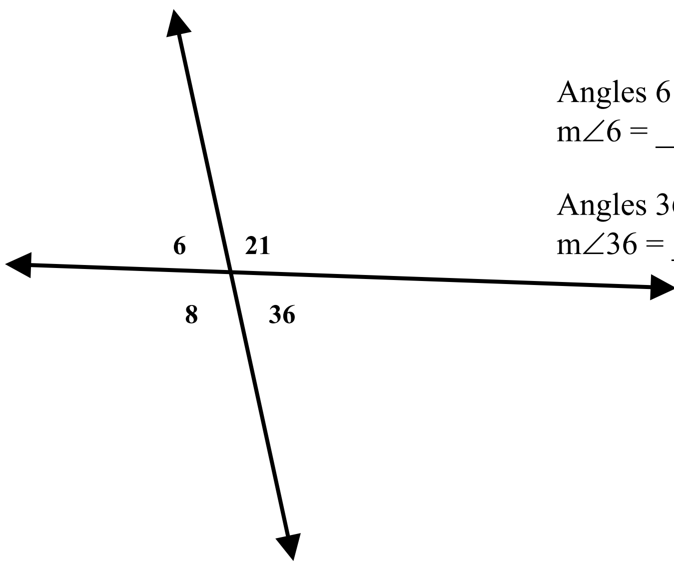
Draw two intersecting lines. Identify a pair of vertical angles and measure them. Does your definition "hold water"? ____________________ Does the relationship you found between the measures of vertical angles "hold water"?_________________________

Draw another pair of intersecting lines and identify vertical angles and measure them. Be prepared to justify your definition of vertical angles.

$\angle 1$ and $\angle 3$ are supplementary angles. Measure them. m$\angle 1$ = _______ m$\angle 3$ = ________

$\angle 4$ and $\angle 3$ are supplementary angles. Measure them. m$\angle 4$ = _______ m$\angle 3$ = ________





Angles 6 and 21 are supplementary angles. Measure them.
m$\angle 6$ = _______ m$\angle 21$ = ________

Angles 36 and 8 are supplementary angles. Measure them.
m$\angle 36$ = _______ m$\angle 8$ = ________

What seems to be the relationship between the measures of supplementary angles? __________________

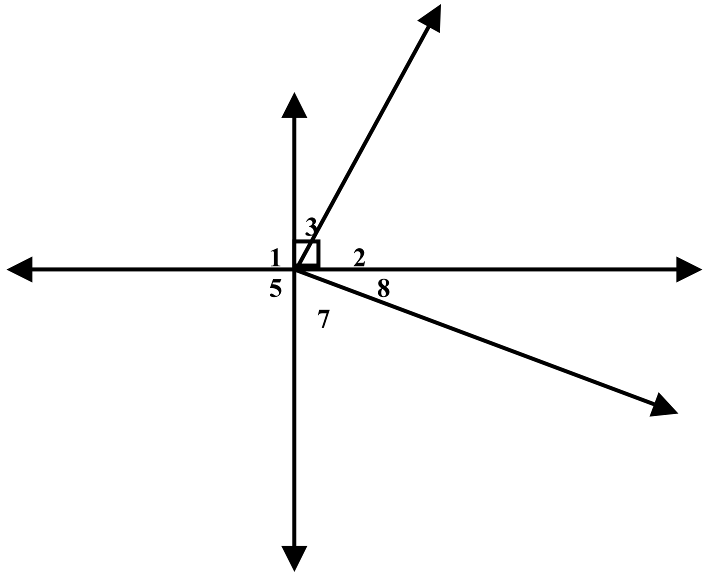
Can you identify supplementary angles? Write a descriptive definition of supplementary angles. Let your teacher read your description and comment on it.

Draw two intersecting lines. Identify three pairs of supplementary angles and measure them. Does your definition "hold water"? _____________________ Does the relationship you found between the measures of supplementary angles "hold water"?___________________

Do supplementary angles have to share a vertex? ____________________
Why or why not?____

Draw another pair of intersecting lines and identify supplementary angles and measure them. Be prepared to justify your definition of supplementary angles.

$\angle 2$ and $\angle 3$ are complementary angles. Measure them. m$\angle 2$ = _______ m$\angle 3$ = ________
$\angle 7$ and $\angle 8$ are complementary angles. Measure them. m$\angle 7$ = _______ m$\angle 8$ = ________



What seems to be the relationship between the measures of complementary angles? _______________

Can you identify complementary angles? Write a descriptive definition of complementary angles. Let your teacher read your description and comment on it.

Draw two perpendicular lines. Identify three pairs of complementary angles and measure them. Does your definition "hold water"? ____________________ Does the relationship you found between the measures of complementary angles "hold water"?________________________ Do complementary angles have to share a vertex? ____________________________________Why or why not?

Draw another pair of intersecting lines and identify complementary angles and measure them. Be prepared to justify your definition of complementary angles.

Pyramids, Prisms, and Cones — Oh, My!

Reporting category	Measurement
Overview	Students investigate and solve problems about volume and surface area of three-dimensional objects.
Related Standard of Learning	8.7

Objectives

- The student will compute the surface area of a pyramid by finding the sum of the areas of the triangular faces and the base
- The student will compute the surface area of a cone by finding the sum of the areas of the side and the base, using formulas
- The student will compute the surface area and volume of rectangular solids (prisms), cylinders, cones and square pyramids, using formulas.

Materials needed

- "Pyramid Scheme" cards, one set per group
- Ruler
- Scissors
- Tape
- Protractor
- 5-by-8 index cards

Instructional activity

1. Prepare a set of "Pyramid Scheme" cards for each small group of students. Each group should have access to index cards, a ruler, scissors, tape, and a protractor.
2. In playing Pyramid Scheme, each player receives a "Pyramid Scheme" card. Each card contains part of the directions for constructing a pyramid. The player holding Card #1 reads the directions and the group works together to complete that part of the construction. Then the player holding Card #2 reads and the group continues to build their pyramid.
3. As each group completes its pyramid, have the participants write a plan for calculating the surface area and volume of the pyramid. The teacher should check the plan. Then students calculate the surface area and volume of the pyramid.

Sample assessment

- What happens if you make each face twice as tall (double the height)? Explain your thinking. Show your calculations.
- How did you measure the height of the pyramid?
- How much taller should each side be to double the volume? Justify your answer. Show your calculations.

Follow-up/extension

- Three groups work together. Will any three identical pyramids fit together to make a rectangular prism?

Homework

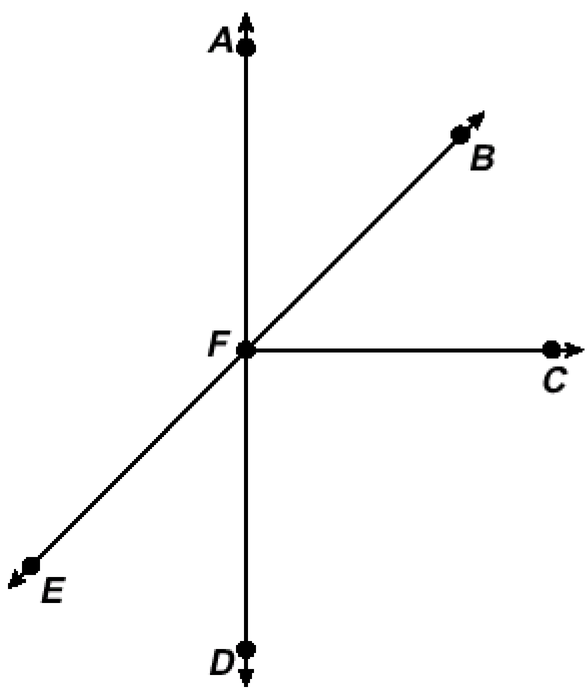
- Compare and contrast the formulas for volume of a pyramid and volume of a cone. Provide written explanations.

Pyramid Schemes

Card 1: Make two identical isosceles right triangles. The "like" sides in the triangles should each be 10 cm long, and they should meet in a right angle. The hypotenuse – the long side – of the triangles should end up about 14.1 cm long. Next, work with your group to assemble your pieces into a single pyramid. Then, work with your group to determine its volume and surface area. Hint: Your two triangles should be connected along one of their 10 cm edges.	**Card 2:** Make a square 10 cm on a side. Use card stock or a big index card. Next, work with your group to assemble your pieces into a single polyhedron. It will have five faces. Assemble your polyhedron with tape. Or, if you want, you can make your triangle with tabs on the edges for gluing. Then, work with your group to determine its volume and surface area. When you are done, work with two other groups to see if you can fit your three pyramids together into a cube.
Card 3: Make a right triangle. One of the sides on the right angle will be 10 cm long. The other will be 14.1 cm long. (Measure carefully, and be sure the right angle is really a right angle!) The hypotenuse – the long side opposite the right angle – will be about 17.3 cm long. Next, work with your group to assemble your pieces into a pyramid with a square base. Then, work with your group to determine its volume and surface area. Your group will need rulers, pencils, tape, card stock, and scissors.	**Card 4:** Make a right triangle. One of the sides on the right angle will be 10 cm long. The other will be 14.1 cm long. (Measure carefully, and be sure the right angle is really a right angle!) The hypotenuse – the long side opposite the right angle – will be about 17.3 cm long. Next, work with your group to assemble your pieces into a single polyhedron. Formula for the volume of a pyramid: $V = Bh/3$, where B is the area of the base and h is the height of the pyramid.

Sample released test items

In the diagram below, $\overleftrightarrow{AD}$ is perpendicular to $\overrightarrow{FC}$.



Which pair is complementary?

A $\angle AFB$ and $\angle BFC$
B $\angle AFB$ and $\angle BFD$
C $\angle AFB$ and $\angle EFA$
D $\angle BFC$ and $\angle CFD$

If ∠QRS and ∠XYZ are supplementary, which *must* be true?

F The sum of the measures of the angles is 90°.

G The sum of the measures of the angles is 180°.

H Both angles can measure between 90° and 180°.

J Both angles must measure less than 90°.

Which of the following is *always* true?

A If two angles are vertical angles, the sum of their measures is 90°.

B If two angles are vertical angles, the sum of their measures is 180°.

C If two angles are vertical angles, one measures more than 90° and one measures less than 90°.

D If two angles are vertical angles, each has the same measure.

If ∠QRS and ∠XYZ are complementary, which *must* be true?

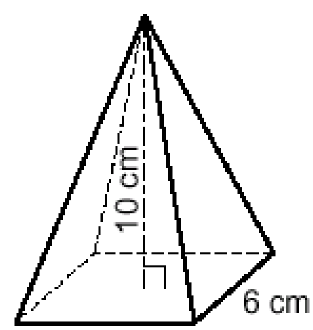
A One of the angles can measure between 90° and 180°.

B The sum of the measures of the angles is 90°.

C The sum of the measures of the angles is 180°.

D Both angles must measure more than 90°.

What is the volume of the square-based pyramid shown below?



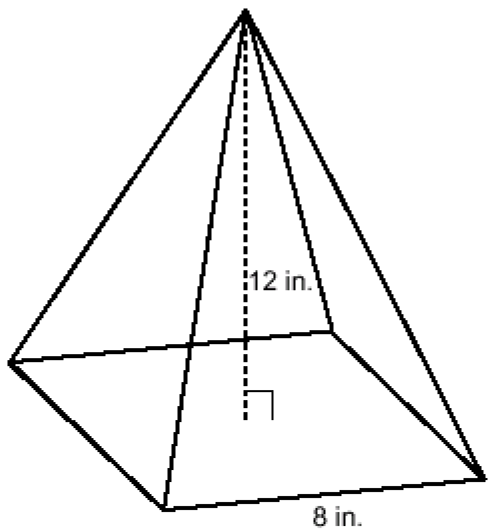
F 20 cm^3

G 60 cm^3

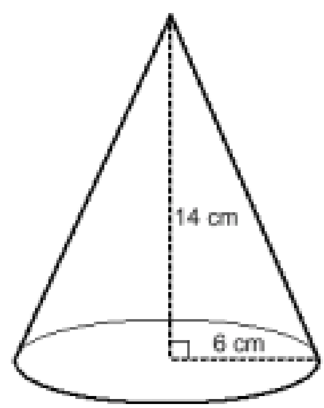
H 120 cm^3

J 360 cm^3

What is the volume of the square-based pyramid shown below?

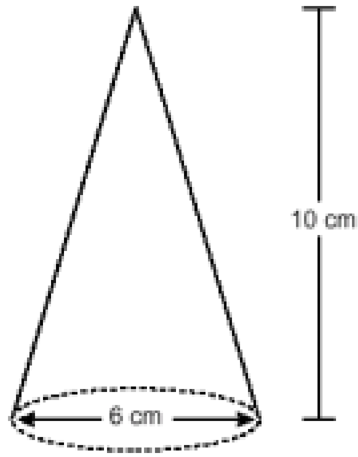


F 96 cu in.
G 256 cu in.
H 384 cu in.
J 768 cu in.



Which is *closest* to the volume of the cone shown above?

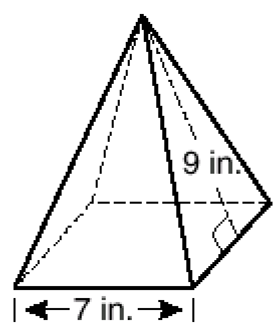
A 87.9 cm^3
B 395.6 cm^3
C 527.5 cm^3
D 1582.6 cm^3



Which of the following is *closest* to the volume of this circular cone?

A 94.2 cm^3
B 282.6 cm^3
C 376.8 cm^3
D 1,130.4 cm^3

As part of an art project, Billy has to paint the surface area of a square-based pyramid. The pyramid has the dimensions shown.



What is the total surface area of the pyramid?

F 441 sq in.
G 301 sq in.
H 252 sq in.
J 175 sq in.

Organizing Topic Geometry

Standards of Learning

8.8 The student will apply transformations (rotate or turn, reflect or flip, translate or slide, and dilate or scale) to geometric figures represented on graph paper. The student will identify applications of transformations, such as tiling, fabric design, art, and scaling.

8.9 The student will construct a three-dimensional model, given the top, side, and/or bottom views.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Identify the geometric transformations (rotation, reflection, translation, and dilation) by using a variety of real-life examples.	______________________
• Demonstrate the reflection of a figure over a vertical or horizontal line on a coordinate grid.	______________________
• Demonstrate 90° 180°, 270°, and 360° rotations of a figure on a coordinate grid.	______________________
• Demonstrate the translation of a figure on a coordinate grid.	______________________
• Construct three-dimensional models, given top, side, and bottom views.	______________________

Transformationally Speaking

Reporting category	Geometry
Overview	Students investigate transformations within the context of tessellations.
Related Standard of Learning	8.8

Objectives

- The student will identify the geometric transformations (rotation, reflection, translation, and dilation) by using a variety of real-life examples.
- The student will demonstrate the reflection of a figure over a vertical or horizontal line on a coordinate grid.
- The student will demonstrate 90-, 180-, 270-, and 360-degree rotations of a figure on a coordinate grid.
- The student will demonstrate the translation of a figure on a coordinate grid.

Materials needed

- Paper (preferably card stock)
- Scissors
- Rulers
- 3 in-by-3 in square of card stock
- Tape
- Large piece of paper (11-by-14)
- Colored pencils
- Crayons or markers
- “Transformationally Speaking,” one copy for each student

Instructional activity

1. Review the transformations (reflections, rotations, dilations, translations) with students. Review the different types of triangles (scalene, isosceles, equilateral, equiangular, acute, right, obtuse) with students. Remind students that to tessellate a plane means to completely cover a surface with a pattern of shapes with no gaps and no overlaps.
2. Since a triangle is the simplest polygonal shape, have students start the investigation of tessellating polygons with triangles. Review the definition of congruent triangles.
3. Pose the following questions for investigation by students. “Which triangle, if any, tessellate? If some triangles tessellate, do they all tessellate?”
4. Distribute the handout to the students, and have them complete each part.

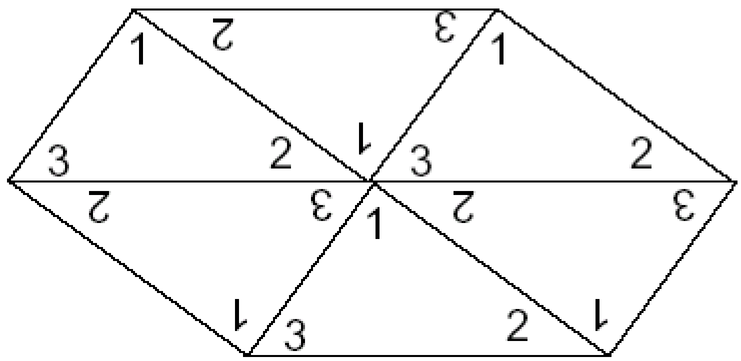
Sample assessment

- Have the students explain why the tessellation by translation works. Have them explain why the tessellation by rotation works.

Transformationally Speaking

Part I: Tessellating triangles

1. Start the investigation with scalene triangles. Use your ruler to draw a scalene triangle. Cut the triangle out. Label the 3 angles as 1, 2 and 3. Copy the triangle several times, cut it out, and label the angles. On a separate piece of paper or card stock, draw a vertex point in the center of the paper. Can you use 6 congruent scalene triangles to completely fill the space around the common vertex point you have drawn? Will any scalene triangle tessellate the plane? Why or why not?



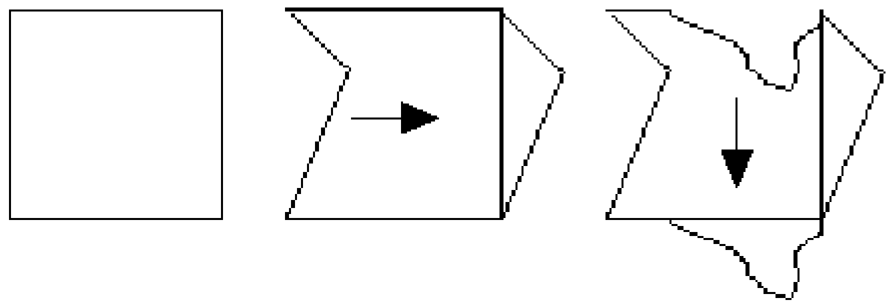
2. Try to make a tessellation using congruent right triangles. Do you think that these triangles will tessellate the plane? Why or why not? Will all right triangles tessellate the plane? Why or why not?
3. Can you make a tessellation using six congruent equilateral triangles? Six congruent isosceles triangles?
4. Can any type of triangle be used to tessellate the plane? Justify your answer.

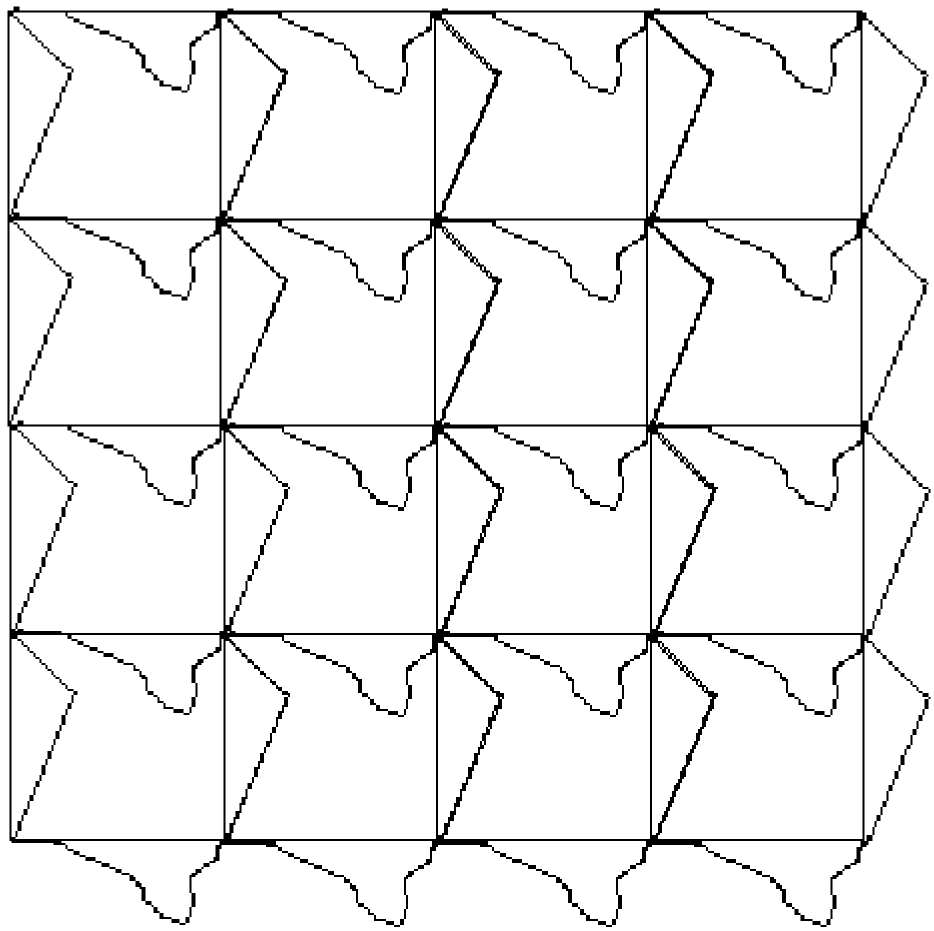
Part II: Tessellating quadrilaterals

1. Design an experiment requiring tessellation of quadrilaterals similar to the Part I experiment with triangles. Ask your teacher to approve your experimental design and data collection plan. Then conduct your experiment and write up your analysis. Be sure to use complete sentences in your writing.

Part III: Tessellations by translation

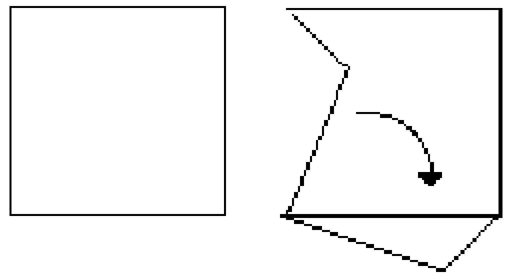
1. Sketch a shape extending into a square from one side. Cut out the shape and slide it across to the other side. Tape it into place.
2. Sketch a shape extending into the square from the top. Cut out the shape and slide it down to the bottom. Tape it into place.
3. Now you have your pattern to tessellate. Trace your pattern repeatedly on your large piece of paper. Color your tessellation and name it.

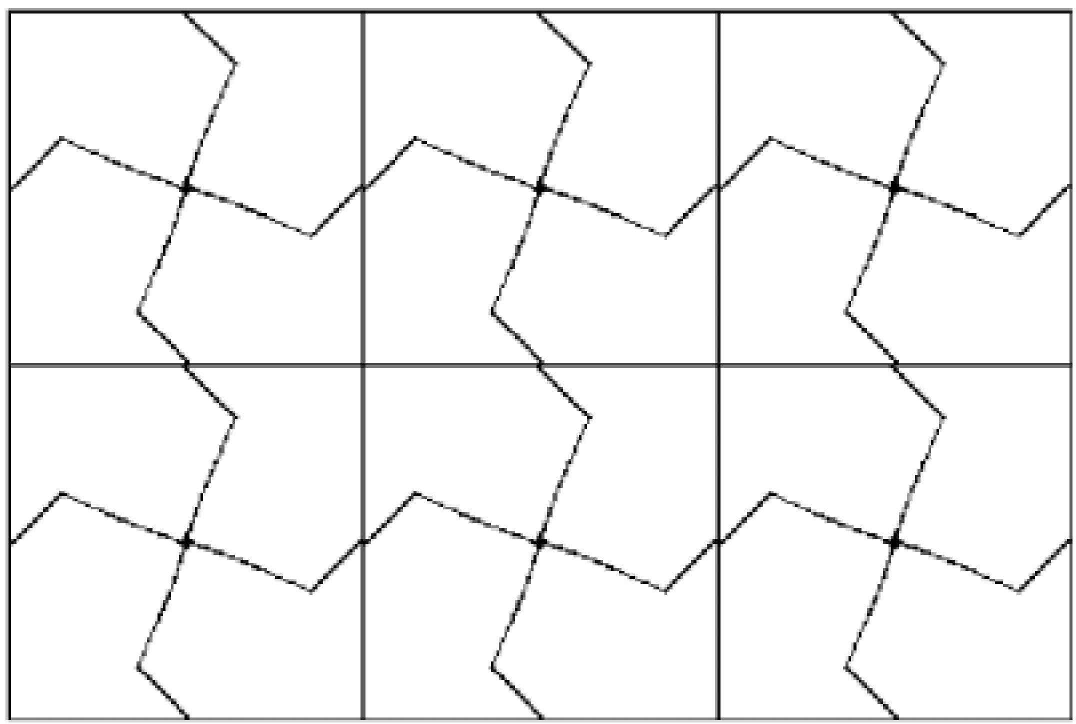




Part IV: Tessellations by rotation

1. Sketch a shape extending into a square from one side. Cut out the shape and rotate it 90º clockwise to the adjacent other side. Tape it into place.
2. Now you have your pattern to tessellate. Starting in the upper left corner of your paper, trace your pattern repeatedly on your large piece of paper. Color your tessellation and name it.





2-D to 3-D and Back Again

Reporting category Measurement

Overview The student will construct three-dimensional models, given top, side, and bottom views.

Related Standard of Learning 8.9

Objective

- The student will construct three-dimensional models, given top, side, and bottom views.

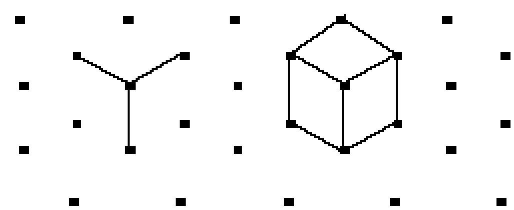
Materials needed

- Isometric dot paper, two or three pieces per student
- Soma pieces (shapes that can be made by combining four cubes)
- 1-inch cubes, ten per student

Instructional activity

Part I: Cubes on dot paper

1. Distribute isometric dot paper to the students. Ask students to compare and contrast isometric dot paper with regular graph (grid) paper. Ask, “What does the prefix *iso* mean? How does this apply to the design of isometric dot paper?”
2. On an overhead, demonstrate how to draw a single unit cube.
3. Show students how to position one of the seven Soma pieces on a diagonal so that students can draw the Soma piece on isometric dot paper.



Sample assessment

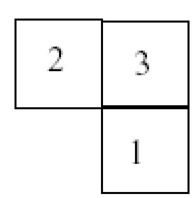
- Have students work in groups to draw all seven Soma pieces on isometric dot paper. Each student should complete his or her own drawing.

Follow-up/extension

- Challenge students to design a structure using all seven Soma pieces. Draw the structure on isometric dot paper.

Part II: Views of cubes

1. Give each student 10 one-inch cubes. Ask students to build a structure that stacks six cubes as shown on the right. This is the view from the top of the structure.
2. Students should then draw the top view, front view, and right-side view. Discuss the drawings and check for accuracy.



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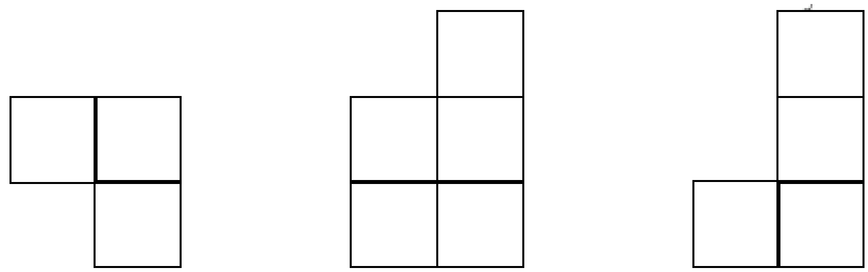


3. Have students work in pairs. Each student should build a structure with some if not all of the cubes and then draw the three views — top, side, and front. Student partners should then exchange drawings and build the structures. Partners should check each other for accuracy.

Sample assessment

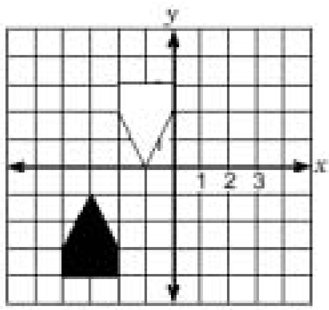
- Have students pairs exchange drawings with other student pairs and work together to build the pictured structure.

Follow-up/extension

- Challenge students to build a new structure using all 10 cubes and draw the three views.

Sample released test items

In which graph is the white figure a reflection of the dark figure over the x-axis?

F


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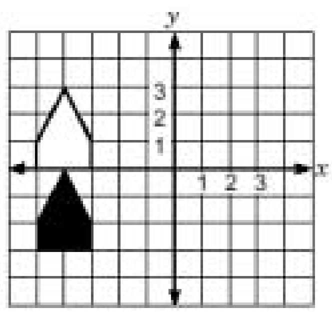

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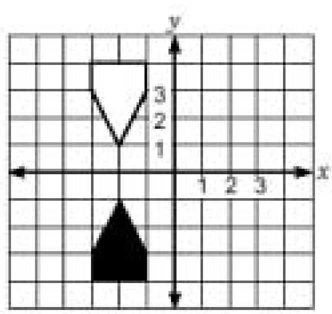

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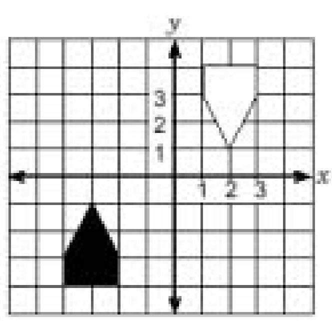
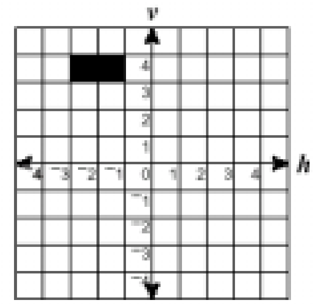
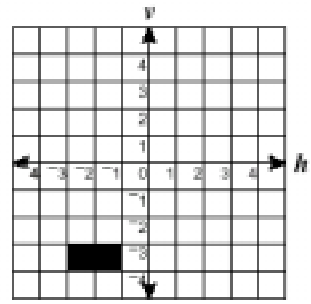
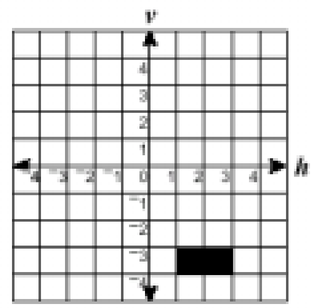
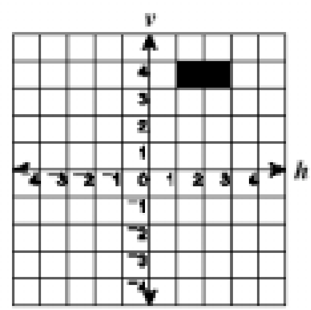


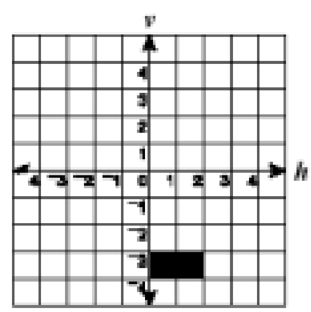


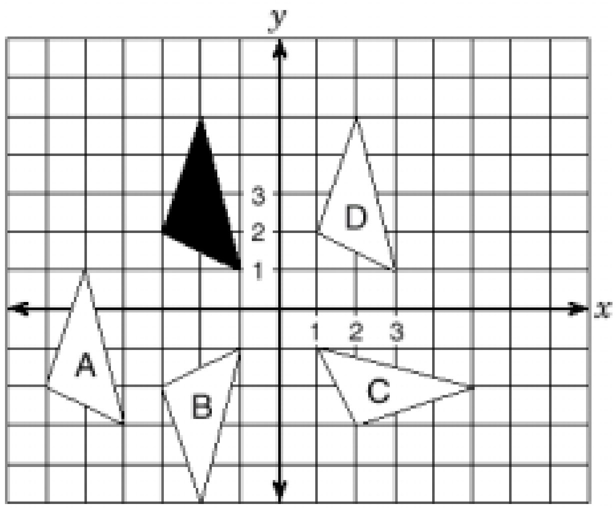
The dark rectangle is reflected over line v. Which shows this reflection?

F


G


H


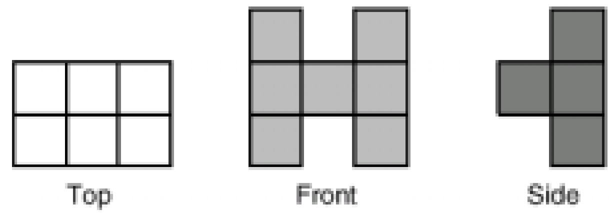
J




Which white triangle shows where the black triangle would be if reflected across the *x*-axis?

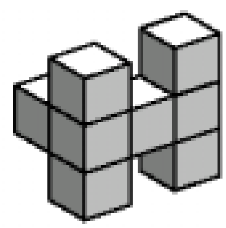
F A
G B
H C
J D

This shows 3 different views of a three-dimensional figure made from cubes.

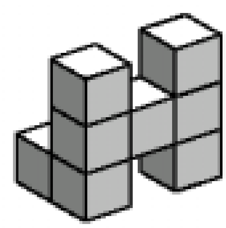


Which could be a drawing of the figure?

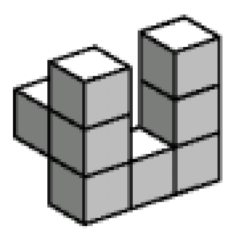
F



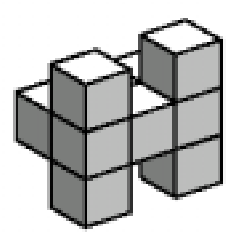
G



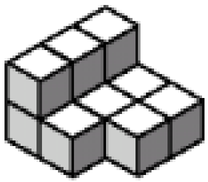
H



J

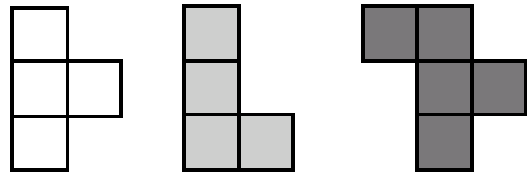


Each cube in this stack has a volume of 1 cubic unit, and each face of those cubes has an area of 1 square unit.



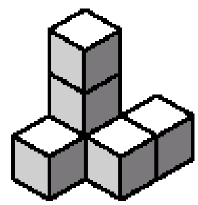
Which could be the surface area of this stack of cubes?

F 18 sq units
G 24 sq units
H 29 sq units
J 36 sq units

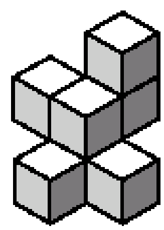


This shows 3 different views of a three-dimensional figure constructed from cubes. Which could be this figure?

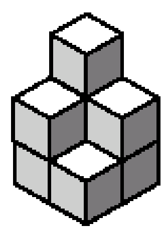
F



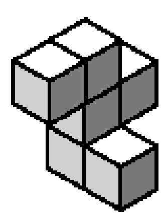
G

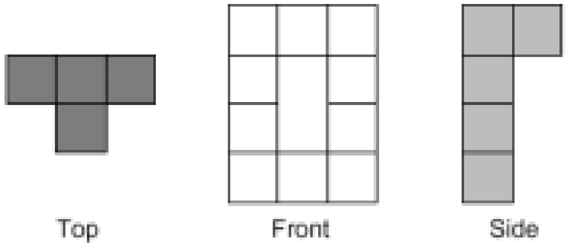


H



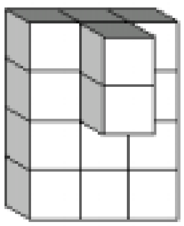
J



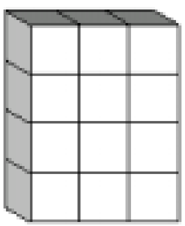


This shows 3 different views of a three-dimensional figure constructed from cubes. Which could be this figure?

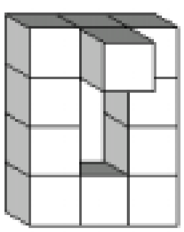
A



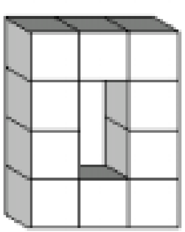
B



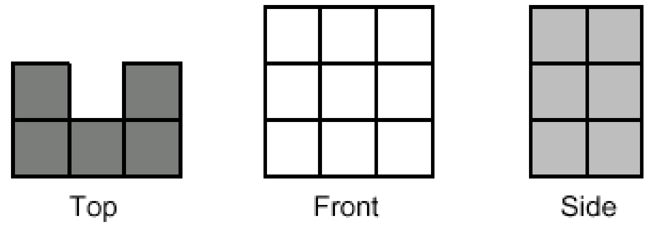
C



D

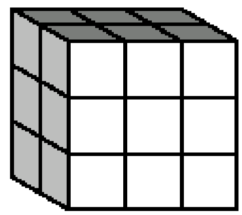


This shows three different views of a three-dimensional figure constructed from cubes.

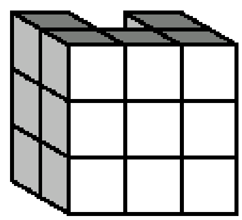


Which of the following could be the figure?

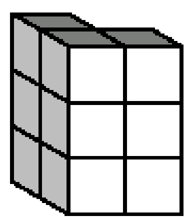
F



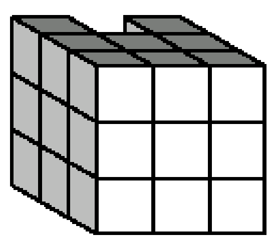
G



H



J



Organizing Topic Probability and Statistics

Standards of Learning

8.11 The student will analyze problem situations, including games of chance, board games, or grading scales, and make predictions, using knowledge of probability.

8.12 The student will make comparisons, predictions, and inferences, using information displayed in frequency distributions; box-and-whisker plots; scattergrams; line, bar, circle, and picture graphs; and histograms.

8.13 The student will use a matrix to organize and describe data.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Analyze a problem situation, and determine the likelihood of an event occurring, using knowledge of probability.	____________________
• Predict the outcome of an event by analyzing its probability.	____________________
• Explain the consequences of making different choices, using knowledge of probability.	____________________
• Make predictions about the outcomes of games of chance, board games, and grading scales by using knowledge of probability.	____________________
• Make comparisons, predictions, and inferences, given data sets of no more than 20 items that are displayed in frequency distributions; box-and-whisker plots; scattergrams; line, bar, circle, and picture graphs; and histograms.	____________________
• Describe the characteristics of a matrix, including designating labels for rows and columns.	____________________
• Use a matrix of no more than 12 entries to organize and describe a data set.	____________________
• Identify the position of an element by row and column.	____________________
• Transfer data from a chart to a matrix.	____________________

The Game Show Problem

(From Problems with a Point*: January 15, 2002 © EDC 2002)*

Reporting category Probability and Statistics

Overview Students simulate a game show to predict outcomes.

Related Standard of Learning 8.11

Objectives

- The student will analyze a problem situation, and determine the likelihood of an event occurring, using knowledge of probability.
- The student will predict the outcome of an event by analyzing the probability.
- The student will explain the consequences of making different choices, using knowledge of probability.
- The student will make predictions about the outcomes of games of chance, board games, and grading scales by using knowledge of probability.

Instructional activity

1. A game show host presents three boxes to a contestant. "In one of these boxes are the keys to a new car!" she says. "Which one would you like to open?"
2. After the contestant makes his choice, the host opens one of the other boxes, which she already knows is empty. "There are only two boxes left, and you've said which one you want to open. Would you like to change your mind and open the other?"

> Assume the host *always* gives the choice — she isn't doing it only when the contestant has picked the correct box!

3. Suppose that after contestant made his choice — and before the host opened one of the boxes — the host asked the contestant if he wanted to switch. Would switching affect his chances of winning? Explain.
4. Now consider what happens after the host shows an empty box. What's your gut impulse — do you think the contestant should open the first box he picked, or open the other?
5. Test your impulse with a simulation with a partner. Get three pieces of paper, about the same size, and a fourth that's much smaller. The small paper will represent the keys and the others will be the box. While one of you looks away (no peeking), the other should hide the small paper with one of the larger ones. Put the other large pieces out so the guesser won't know where the small piece is. (The "host" should be sure to remember where, though!) Play the game several times, sometimes changing and sometimes not. Each time, record whether the guesser changed or not, and also whether the guesser won. After a few games, change roles so both partners can be the guesser.

> You could write "keys" or "car" on the small paper, if you want — but do it lightly, so it can't be seen through the paper covering it! Another option is to use three playing cards, two of one kind (such as Aces or jokers) and one of a different kind, to represent the inning box.

   a. What percentage of the games in which the guesser changed won the car?
   b. What percentage of the games in which the guesser did *not* change won the car?
6. Combine your results with those of the rest of the class. What percentage of all the games in which the guesser changed did the guesser win? What percentage of those in which the guesser didn't change were winners?

7. Would you advise the contestant to open the first box picked, or switch and open the remaining box after the host opens one?
8. You answered this question by doing several trials of a simulation.
   a. Explain why this was a simulation.
   b. Is your result correct, beyond a doubt? Or is it possible that if you did many more trials, you'd find that you gave the contestant bad advice? Explain.

Teacher notes

- Hint for problem 5: Separate the games in which the guesser changed from the games in which the guesser did not. The percentage won by changing is

$$\frac{\text{number of games won by changing}}{\text{total number of games in which the guesser changed}}$$

- Hint to problem 8a. What does *simulate* mean?
- Hint to problem 8b. What should happen to the experimental probability (what you calculated in problems 2 and 3) as you conduct more and more trials?
- This classic problem (often called the "Monty Hall Problem," after the game show host on *Let's Make a Deal)* has generated a large amount of discussion, including among mathematicians, since many people consider the result somewhat paradoxical. (If there are ultimately two boxes to choose from, it might seem the contestant should have a 50 percent chance with either box.) Some people may disagree with the results given below, but most will agree that this is sound reasoning. Hopefully, the students' simulations will support the results. The original pick has essentially two results: choose the keys ($\frac{1}{3}$ chance) or an empty box ($\frac{2}{3}$ chance). If the box with the keys is picked, the host can show either box.

> You can find more about this problem in the following references:
>
> L. Gillman. "The car and the Goats." *American Mathematical Monthly*, January 1992, pp. 3–7.
>
> E. Barbeau. "Fallacies, Flaws, and Flimflams." *The College Mathematics Journal*, March 1993. (Identifies 63 articles about this problem and its variations.)
>
> The "Monty Hall Problem" Web site, math.rice.edu/~ddonovan/montyurl.html

- Opening the original box results in a win, opening the other results in a loss. So from this case, the contestant has a $\frac{1}{3}$ chance to win by keeping the same box. (Equivalently, the contestant has a $\frac{1}{3}$ chance to lose by changing boxes.) If an empty box is picked, the remaining empty box will be shown. Opening the original box results in a loss, opening the other results in a win. So from this case, the contestant has a $\frac{2}{3}$ chance to lose by keeping the same box. (Equivalently, the contestant has a $\frac{2}{3}$ chance to win by changing boxes.)
- You can put all this together in various ways. For example, if the contestant plays several times and always switches, he should win about $\frac{2}{3}$ of the time and lose about $\frac{1}{3}$ of the time. Or, the chance to win is $\frac{1}{3}$ if he chose correctly and kept the box, or $\frac{2}{3}$ if he chose incorrectly but switched. However you consolidate the information, it seems clear that the contestant should switch.

Well, What Did You Expect?

(From Problems with a Point*: November 6, 2001 © EDC 2001)*

Reporting category Probability and Statistics

Overview Students simulate a game of chance and investigate possible outcomes.

Related Standard of Learning 8.11

Objectives

- The student will analyze a problem situation, and determine the likelihood of an event occurring, using a knowledge of probability
- The student will predict the outcome of an event by analyzing the probability
- The student will explain the consequences of making different choices, using knowledge of probability
- The student will make predictions about the outcomes of games of chance, board games, and grading scales by using knowledge of probability.

Materials Needed: 3 pieces of paper about the same size and another piece of paper that is much smaller

Instructional activity

1. Three friends are playing a game. There are six marbles in a bag: 1 red, 2 white, and 3 blue. The players choose different colors before playing, so one player is "red," one is "white," and the other is "blue."
2. To play, they each pull out a marble from the bag. If the red player pulls out a red marble, he gets 3 points. If the white player pulls out a white marble, she gets 2 points. The blue player gets 1 point for pulling out a blue marble. The first to get 6 points wins the game.

> Of course, after finding out if he or she scored, the player puts the marble back in the bag.

3. The friends weren't sure if the scoring system was fair. They decided to test whether the system was fair. First, the red player took 60 turns, one after the other.
   a. What's the probability that the red player pulls out a red marble on any given turn?
   b. On 60 turns, how many times would you *expect* the player to pull out a red marble?
   c. How many points would you expect the red player to get with 60 turns?
   d. How many points, *on average*, would a single turn give the red player?

> Because this is an average, it may not actually be possible to get that many points on a single turn. For example, you might only be able to earn whole number grades for a test — but if your grades are 80, 83, and 90, your average is $84\frac{1}{3}$.

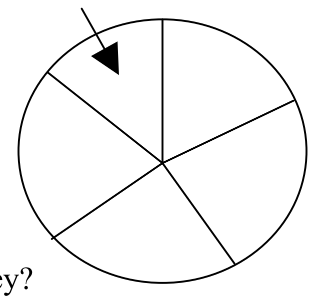
4. How many points would you expect the following players to get on a single turn, on average?
   a. The blue player
   b. The white player
5. Is this a fair game? Explain.
6. When you have a probability situation that involves numbers (like the score of the game), the *expected value* of the situation is the value you can expect, on average, to get for a single instance

of the situation. Your answers to problems 1d and 2 are the expected values for the players' scores on a single turn.

Sample assessment

- Suppose a fourth person joins the game. They add black marbles to the bag. The probability of drawing a black marble from the bag is p, and the new person scores s points for pulling a black marble out.
  a. On 60 turns, how many times would you expect to pull out a black marble?
  b. What would the black player's score be after 60 turns?
  c. How many points on average would the black player get on a single turn?
  d. Would you get the same answer if you used a different number of turns (say 120, or 392)? Why or why not?
- Your answer to problem c is a formula for finding the expected value for a single event (like pulling out a black marble). Use your formula to answer the following questions.

Follow-up/extension



- For a school fundraiser, a person spins the wheel shown here. (The spaces are all of equal size.) If it stops with the arrow pointing on the white space, the person wins a $10 gift certificate to MusicWorld. If it stops elsewhere, the person gets nothing.
  a. What is the expected value?
  b. If the people pay $1 to spin the wheel, will the school make any money? (That is, the school wants the game to be unfair, to the school's advantage. Is it?)

Homework

- This situation is a little different. In a board game, a player landing on a "Risk It!" square has an opportunity to roll a standard die (numbered 1–6). A roll of 5 or 6 gets the player an extra 1000 points toward victory. A roll of 1, 2, 3, or 4 costs the player 600 points.

> Remember, the expected value for a situation is what the person would get each turn on average, if he or she played many times.

  a. Ignoring the possibility of losing, what is the expected value that the player will *win*?
  b. Ignoring the possibility of winning, what is the expected value that the player will *lose*?
  c. The expected value, in this case, is the expected amount the player might win *minus* the expected amount the player might lose. Find the expected value. Do you think the player should "risk it"?
  d. Another way to calculate the expected value in this case is to consider playing a certain number of times. (You used this idea in step 3 above.)
     i. If you played 30 times, how many times would you expect to gain points? How many would you gain?
     ii. For 30 tries, how many times would you expect to lose points? How many points would you lose?
     iii. What would you expect for the total effect on your score from playing 30 times? (Would you gain or lose points, and how many?)
     iv. From your previous answer, find the points gained or lost per turn, on average. Does this agree with your answer to part c?

Data Mania

Reporting category Probability and Statistics

Overview Students make comparisons, predictions, and inferences based on various graphical representations.

Related Standard of Learning 8.12

Objective

- The student will make comparisons, predictions, and inferences based on data represented in frequency distributions; box-and-whisker plots; scattergrams; line, bar, circle, and picture graphs; and histograms.

Materials needed

- "Name That Graph," one copy for each student
- "Relationship of Height to Age," scattergram for display
- Graph of relationship of height to age
- *USA Today*, one issue for each group of four students
- Chart paper for each group
- Scissors and glue

Instructional activity

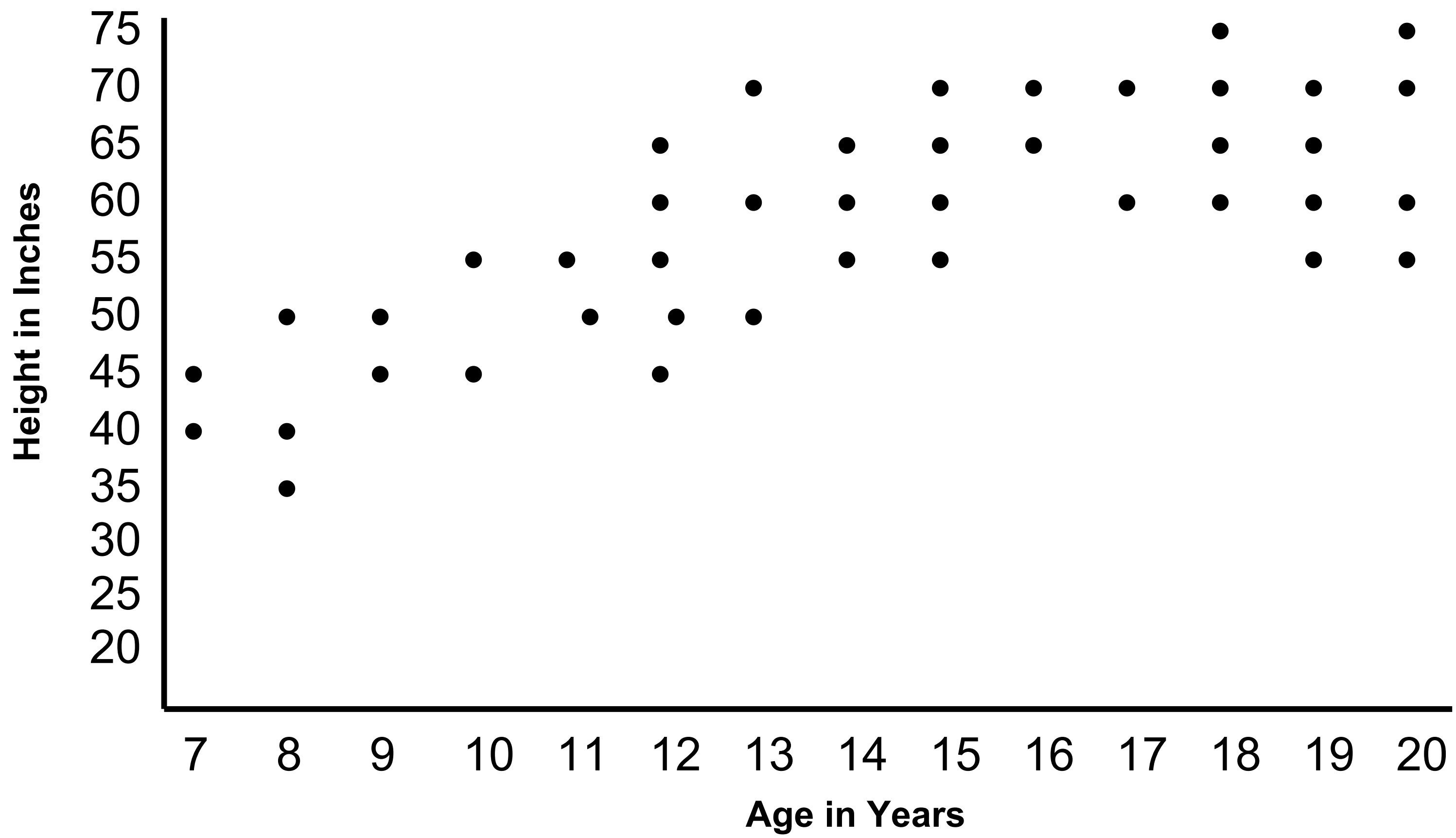
1. Use the "Name That Graph" handout to review quickly the types of graphs that students will need to use in this activity. Have students quickly name the type of graph and give an example of information that could be represented in the graph. A matching exercise could also be used as a quick review of the various types of graphical representations.
2. Explain to students that they will be reviewing a variety of data displays and creating questions from the data for their classmates to answer. Model the process with the sample graph included with this lesson. Display the scattergram "Relationship of Height to Age." Ask students to pose questions that can be answered based on the data represented in the graph. Indicate that the responses to the questions should
   - compare the characteristics of the data
   - predict the results of additional instances of the date (e.g., people older than 21 would be taller than what height)
   - infer information from the data.
3. Record questions on chart for students to use as models. Suggested prompts for questions include:
   - What are the differences in height between students who are 7 years old?
   - What is the typical height of a 12-year-old?
   - What would you predict to be the differences in height (in inches) between the shortest and tallest within any classroom in the school?
   - Based on the height of students in second grade (7-year-olds), what could you infer about the height of a water fountain in primary school versus one found in a high school?
4. Distribute copies of *USA Today* to each group of students and a sheet of chart paper. Explain to students that each group will select one graphical representation of information from the *USA*

Today newspaper. Students will glue the selected graph onto the chart paper. As a group, they will create a summary of the information displayed in the graph and record it under the graph. Then they will compose three questions that can be answered from the displayed data. The questions need to address the comparison of the data, and predictions and inferences based on the data.

5. Have students present the final product to the class, asking their questions that they have composed.

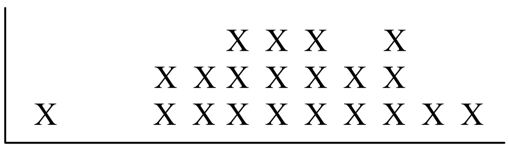
Relationship of Height to Age

(Sample graph)



Name That Graph

1.



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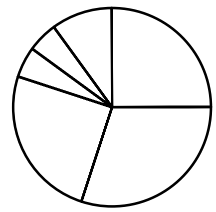
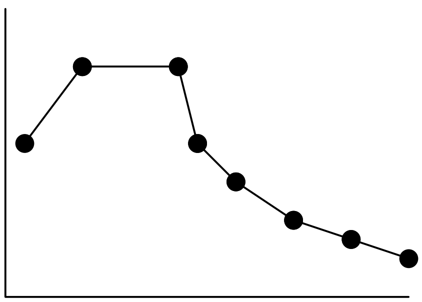


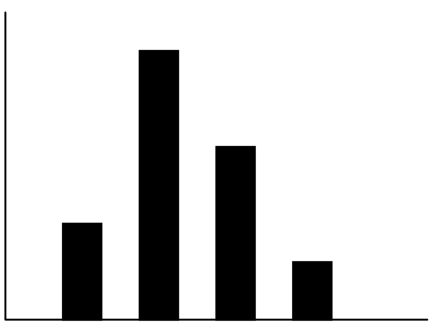
3.



4.

0	3 3 5 7 8 9
1	0 2 3 5 6 6 8 9
2	0 1 3 3 3 5 5 8
3	0 5
4	5

5.



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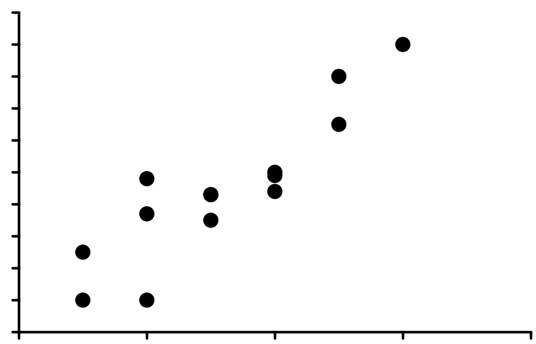
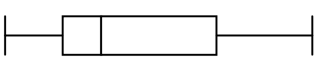
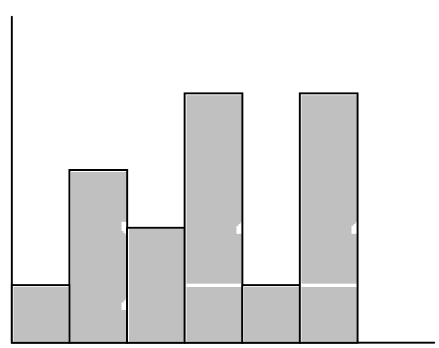


7.



8.



Homework, TV, and Sleep

(From Problems with a Point*: April 19, 2002 © EDC 2002)*

Reporting category Probability and Statistics

Overview Students use data presented in graphic format to make predictions.

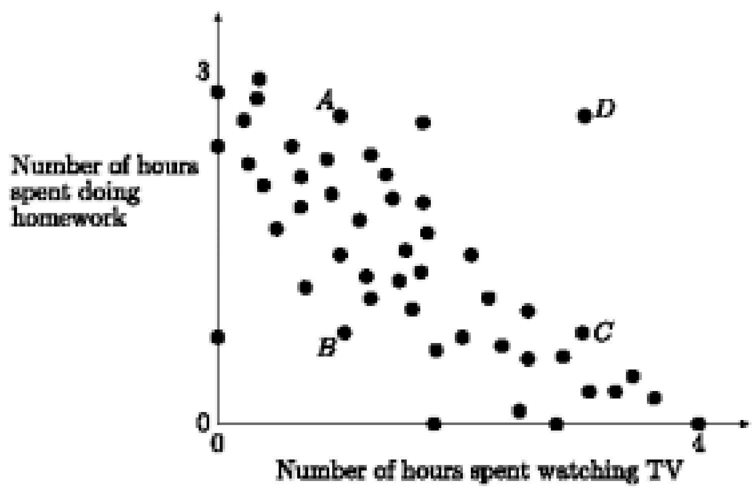
Related Standard of Learning 8.13

Objective

- The student will make comparisons, predictions, and inferences, given data sets of no more than 20 items that are displayed in frequency distributions; box-and-whisker plots; scattergrams; line, bar, circle, and picture graphs; and histograms.

Instructional activity

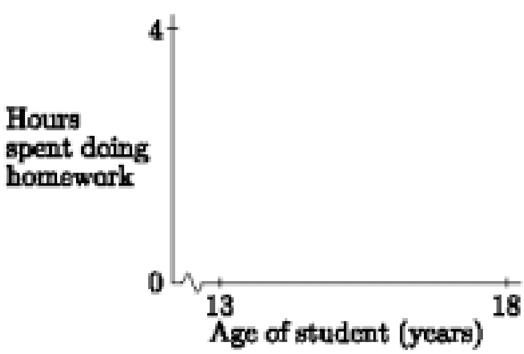
Annie asked a group of teenagers how much time they spent doing homework one evening and how much time they spent watching TV. Here is a scatter plot to show the results.

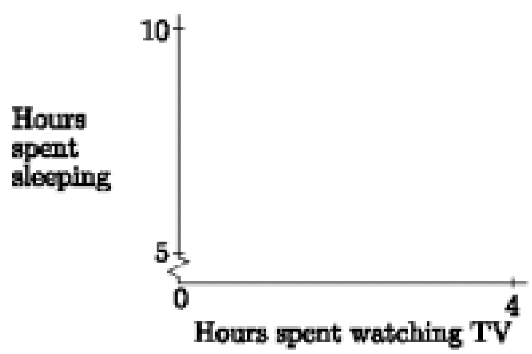


1. Which of the four points, *A*, *B*, *C*, or *D,* represents each of the following statements? Write one letter for each statement.
   a. "I watched a lot of TV last night and I also did a lot of homework."
   b. "I spent most of my evening doing homework. I only watched one program on TV."
   c. "I went out last night. I didn't do much homework or watch much TV."
2. Make up a statement that matches the fourth point.
3. What does the graph tell you about the relationship between time spent TV and time spent doing homework?

Sample assessment

- Annie also drew scattergrams that showed that
  - older students tend to spend more time doing homework than younger students
  - there is no relationship between the time students spend watching TV and the time students spend sleeping.
- On the axes at right, show what Annie's scatter plots may have looked like.





Vashon-Maury Island Soil Study

Organizing topic Using Algebraic Topics

Overview The Vashon-Maury Island soil study provides students with a real application of box-and-whisker plots in data analysis.

Related Standard of Learning 8.12

Objective

- The student will make comparisons, predictions, and inferences, given data sets of no more than 20 items that are displayed in box-and-whisker plots.

Materials needed

- "Vashon-Maury Island Soil Study" handouts, one copy for each student (Sections 1.0, 6.0, and 6.3 of the Final Report, two maps, and two box-and-whisker plots)
- A transparency of each of the two maps (optional)

Instructional activity

Note: This activity is an excellent opportunity to address the question, "When am I ever going to use this?" As a result of this study, Vashon-Maury Island is now an EPA Superfund cleanup site.

1. Distribute copies of the handouts to each student. Have students read Section 1.0 of the Final Report. The maps will be helpful in following the article. You may wish to discuss the implications of lead contamination with students.
2. The two box-and-whisker plots address different elements: arsenic and lead. Divide students into small groups. Assign each group one of the box-and-whisker plots. Using the map and box-and-whisker plot, each group must determine where Zones 1, 2, and 3 are.
3. Have each group include a complete description of their discussions and findings. The reasoning/justification process is critical and must be detailed.

Sample assessment

- Have each group present its conclusions to the whole class.

Homework

- Have the students read Sections 6.0 and 6.3 of the Final Report. Does the information in Sections 6.0 and 6.3 support your group's conclusions? Why or why not?

Final Report of the 1999–2000 Vashon-Maury Island Soil Study

SECTION 1.0

Project description and objectives

In 1999 and 2000, Public Health – Seattle & King County (PHSKC) performed the first comprehensive survey of contamination by arsenic, lead, and cadmium in surficial soils on Vashon-Maury Island, Washington. This study also included an initial "pilot scale" evaluation of surficial soils in shoreline areas of the King County mainland, east of Vashon-Maury Island. The results of soil sampling and analysis in both areas are presented and evaluated in this report.

In this study, soils were sampled to a depth of 6 inches below the forest duff layer. The term "surficial" rather than the simpler term "surface" soils is used to emphasize that there can be important differences in potential exposures (e.g., frequency of contact) between soils within the top inch or so versus soils at depths of, for example, 4 to 6 inches. "Surficial" is in this sense used to refer to true surface soils as well as near-surface soils.

Previous studies

Since the early 1970s, more than a dozen studies have determined arsenic concentrations in surficial soils in selected areas of Vashon-Maury Island (see Sections 2.2 and 7.0, and the references in Section 9.0). Analyses in some of the studies included lead, with analyses of additional metals (e.g., cadmium, selenium, or antimony) occasionally reported. Those studies were motivated by concerns over the magnitude, extent, and fate of contamination resulting from operation of the former ASARCO Tacoma Smelter, located at Ruston on the shoreline of Commencement Bay, a few miles south/southwest of Vashon-Maury Island (see PSAPCA 1981a and 1981b for a general review of Tacoma Smelter operations and emissions). The Tacoma Smelter operated for over 90 years, closing smelting operations in 1985 and arsenic processing operations in 1986. For many years, the Tacoma Smelter was the sole domestic supplier of arsenic for the United States. Arsenic has generally been considered the best tracer element for evaluating smelter impacts (see, for example, Crecelius et al. 1974 and PSAPCA 1981a and 1981b), and it has also been the focus of concerns for potential human health risks from smelter emissions. The smelter site and surrounding areas (within an approximate one-mile radius) are subject to ongoing cleanup actions under EPA's Superfund program.

Soil studies over the past 30 years, performed both during smelter operations and after closure, have documented elevated levels of arsenic, lead, and other metals in surficial soils on Vashon-Maury Island. However, the previous studies in aggregate do not provide a comprehensive portrait of current soil contamination levels. Separate studies used different protocols for collection and laboratory analysis of soil samples, making comparisons among studies more difficult. The studies also targeted different types of land uses, such as residential yards and garden areas, parks and playgrounds, and relatively undeveloped and undisturbed areas. The degree of soil disturbance has been shown to be an important factor affecting the residual concentrations of air-deposited contaminants in surficial soils; the marked differences in past soil-disturbing activities among sampled land types is expected to contribute significantly to variability in results. The representativeness of studies performed up to 30 years ago for characterizing current soil contamination levels can be questioned, especially with respect to current depth profiles for contamination. Finally, the number (density) and spatial pattern of prior sampling locations are both limited; most of the previous studies have focused on Maury Island and south Vashon Island, with relatively few samples collected on north Vashon Island. Evaluation of all of these factors supported the conclusion that previous studies, considered cumulatively, could not provide a comprehensive description of current soil contamination levels on Vashon-Maury Island. Previous study results were used in developing the study design for the current PHSKC study (see Section 2.2).

In addition to soil sampling and analysis studies, several other types of investigations help to characterize the likely extent of soil contamination in the area downwind of the former Tacoma Smelter. Those additional studies included deposition modeling, plume tracking (opacity) studies, an extended series of precipitation chemistry monitoring studies, sediment core chemistry monitoring, vegetation sampling, and bee biomonitoring (see references listed in Section 9.0). Several of these studies (e.g., deposition modeling, precipitation chemistry studies, and bee biomonitoring) have provided "contour" mapping of potential impact areas. The resulting predicted pattern of soil contamination levels on Vashon-Maury Island had not been fully validated by soil sampling and analysis data prior to this PHSKC study. The "contouring" study results also indicated that the spatial scale for measurable smelter impacts may be on the scale of tens of miles, extending onto the mainland areas of King County.

Very few soil samples from such mainland areas are available from previous studies; nevertheless, some elevated soil arsenic results have been reported. The study of Vashon-Maury Island soils was extended to include an initial "pilot scale" evaluation of soils in mainland King County shoreline areas east of Vashon-Maury Island.

Project description and objectives

The ongoing Superfund cleanup of areas surrounding the Tacoma Smelter that were affected by air deposition from smelter emissions does not include any areas on Vashon-Maury Island (see USEPA, Region 10 1993). Residents of Vashon-Maury Island were active participants in several earlier processes involving Tacoma Smelter operations and impacts, including the PSAPCA Environmental Impact Statement process under SEPA and EPA's proposed rules for arsenic under the National Emissions Standards for Hazardous Air Pollutants (NESHAPS) provisions of the Clean Air Act. After EPA issued its Superfund Record of Decision for cleanup of residential areas in Ruston and North Tacoma, in 1993, soil contamination issues arose for a number of years primarily in connection with real estate transactions. In 1998 and 1999, a proposal to expand gravel mining operations at the Lone Star (now Glacier Northwest) Maury Island gravel mine (see King County DDES 1999) resulted in two independent soil studies on gravel mine property. Both studies showed significantly elevated soil arsenic concentrations, with maximum levels in relatively undisturbed areas exceeding 300 ppm. These findings resulted in heightened interest in soil contamination levels and impacts among some Vashon-Maury Island residents. In this same time frame, Ecology was also beginning to evaluate "area contamination" issues under the Model Toxics Control Act (MTCA), which represented areas of contamination far larger than the typical MTCA site. Widespread soil contamination by arsenic and other smelter-related metals was recognized by Ecology as one such "area contamination" problem.

A recently discovered study by Environment Canada comparing periods before and after smelter closure showed significant decreases in sulfate and arsenic levels in precipitation along the Canadian border coinciding with smelter closure. See D.A. Faulkner, "The Effect of a Major Emitter on the Rain Chemistry of Southwestern British Columbia — a Second Look," presented at the November 8–10, 1987 annual meeting of the Pacific Northwest International Section, Air Pollution Control Association, in Seattle, WA.

The current study was conceived in response to both community concerns specific to Vashon-Maury Island and Ecology interest in evaluating area contamination problems where threats to human health or the environment could occur over large areas. It is intended to provide a comprehensive survey of current surficial soil contamination over all areas of Vashon-Maury Island, as well as an initial evaluation of King County mainland areas to the east. Funding for the study was provided jointly by PHSKC and Ecology. PHSKC staff developed study plans (see PHSKC 1999), in collaboration with Ecology, and provided staff to perform all field sampling work. Laboratory analyses were performed by OnSite Environmental, Inc., Redmond, Washington under contract to PHSKC. The study design was developed through a multi-party Vashon-Maury Island Soils Work Group. PHSKC retained Gregory L. Glass as a consultant to work with the Soils Work Group in developing a study design; he also performed data evaluations and prepared the project report. PHSKC maintained the database of analytical results and performed data validation reviews. Ecology provided technical support and oversight of the study.

The primary objectives of the study are to document the current magnitude and large scale spatial patterns in soil contamination in relatively undisturbed King County areas downwind from the former Tacoma Smelter, including a comprehensive survey of all of Vashon-Maury Island and an initial exploration of King County mainland shoreline areas to the east. This regional-scale study provides information useful for focusing additional investigation and response actions by the agencies and the public. Several detailed data evaluation objectives were also identified and reflected in the study design:

1. to evaluate the vertical pattern of soil contaminant concentrations at different depths (depth profiles);
2. to assess the relationship (correlations) among different soil contaminants; and
3. to evaluate how soil contaminant concentrations vary within relatively small areas (variability and spatial scale)

Since all soil sampling occurred in relatively undisturbed areas, the results are believed to be biased toward upper bound contaminant concentrations. All data interpretations should take this study design feature into account. This study does not provide any information on the comparative magnitudes or patterns of contamination between relatively undisturbed and developed/disturbed properties.

The detailed study design and data evaluations are discussed in subsequent sections of this report. In brief, the study involved collection of soil samples at 0–2 inch and 2–6 inch depth intervals; analyses for arsenic, lead, and cadmium — three primary smelter-related contaminants; targeted sampling in forested areas believed to represent least disturbed soils where contaminant levels are likely to be highest; and collection of samples in an approximate grid pattern within forested areas, plus additional collection of closely spaced samples (in comparison to grid spacing) to look at local variability. In all, 436 soil samples were analyzed for arsenic and lead, and 338 for cadmium.

SECTION 6.0
Data evaluations

As described in Section 1.0, a set of data evaluation objectives was identified for this study. Each of the identified data evaluation objectives is discussed in its own subsection below, preceded by a brief discussion of the approach taken to data evaluations. The discussions that follow emphasize the primary data evaluation results; references are provided to more detailed statistical results and data plots that are included as Attachment D, for readers interested in such details.

SECTION 6.3
Spatial patterns

Arsenic and lead results were mapped using GIS software. Figures 7 and 8 show the resulting maps; spatial sampling locations are identified only at the level of the parcel sampled. Not all results (see Attachment B for a complete listing) are plotted on Figures 7 and 8; only the maximum concentration at any depth is plotted for a given sampling location. Where only one grid location at a parcel was sampled, only a single result is shown. Multiple results are shown when spatial scale sampling occurred or when more than one grid cell location within a single parcel was sampled. For example, at sampling location 113 in Zone 1, at the southern end of Maury Island, the three values of 250 ppm, 210 ppm, and 130 ppm represent the maximum arsenic concentrations regardless of depth at the three spatial scale sampling locations (i.e., B1-113-T, B1-113-TY, and B1-113-SZ). The sampling grid locations are color coded on Figures 7 and 8 to reflect ranges of maximum contaminant concentrations, making it easier to see spatial patterns in the results.

Inspection of Figures 7 and 8 reveals that substantial differences in contamination levels can occur within localized areas. This small scale spatial variability is discussed further in Section 6.6. A large scale spatial pattern is also visible; that large scale pattern is similar for arsenic, lead, and cadmium, indicating a high degree of co-occurrence and correlation among the three contaminants (discussed further in Section 6.4). The highest contaminant levels are generally found on Maury Island (Zone 1B), followed by South Vashon Island (Zone 1A) and the Mainland (Zone 2). The lowest concentrations are found on North Vashon Island (Zone 3). The degree of variability in results within zones is large enough that the distributions in these subareas are broadly overlapping rather than entirely dissimilar. Nevertheless, there is a clear and meaningful large scale spatial pattern evident in the data. (This large scale pattern would be particularly evident if, for example, the results were averaged over blocks of significant size, such as 1 to 2 square miles). As noted in Section 2.3, all mapped results are representative of relatively undisturbed areas and are very likely not generalizable to developed/ disturbed areas.

Land use and the history of site disturbance are emphasized again as very important factors affecting the current pattern (including depth of contamination) and magnitude of residual soil contamination. The best use of the maps of contamination magnitudes from this study is as likely upper bounds (or near upper bounds, because the number of samples is limited) for soil contamination across various types of land uses.

The dominant spatial pattern can be illustrated using the sampling zones to partition the data set (while recalling that the actual pattern is gradational rather than clearly associated with the chosen zone boundaries). Tables 4a, 4b, and 4c provide detailed comparisons of selected percentile values across all sampling zones, based on individual samples; Table 5 provides similar information for maximum location values. The ranking of subareas is shown clearly in these Tables. Bar charts showing the distributions for arsenic and lead in each sampling zone are included in Attachment D. Comparison of these bar charts across zones also reveals the large scale spatial pattern in the results.

Figures 9 through 10 show multiple box-and-whisker plots of arsenic and lead results (with not detected results assigned values of one-half the detection limit) for individual samples by sampling zone. These visual summaries of the statistical distributions by zones show that the median and extreme high values follow the ranking of Zones 1, 2, and 3, from highest to lowest levels; more detailed examination supports the finding that Zone 1B (Maury Island) shows generally higher contaminant levels than Zone 1A (South Vashon Island). [Note: the lead box and whisker plot for Zone 3 (see Figure 10) is likely biased high because there are comparatively fewer 2–6 inch analyses in Zone 3, and lead depth profiles typically show higher concentrations in the 0–2 inch interval. Thus, the actual differences between Zones 1 and 2 versus 3 for lead are probably understated. Additional box-and-whisker plots for arsenic and lead in the 0–2 inch samples only are provided in Attachment D, showing similar results.]

Box and whisker plots provide a visual summary of statistical data distributions. The box encloses the 25th to 75th percentiles of the data (i.e., the middle 50 percent), defining the size of the interquartile range. The median is shown as a line within this box. The straight line whiskers above and below the box extend to the values in the data set closest to, but not exceeding, 1.5 times the interquartile range above and below the 75th and 25th percentiles. Outlier values beyond the whiskers are plotted individually, with a special symbol if they are more than 3 times the interquartile range above or below the ends of the box. Multiple box and whisker plots allow data sets to be rapidly compared visually.

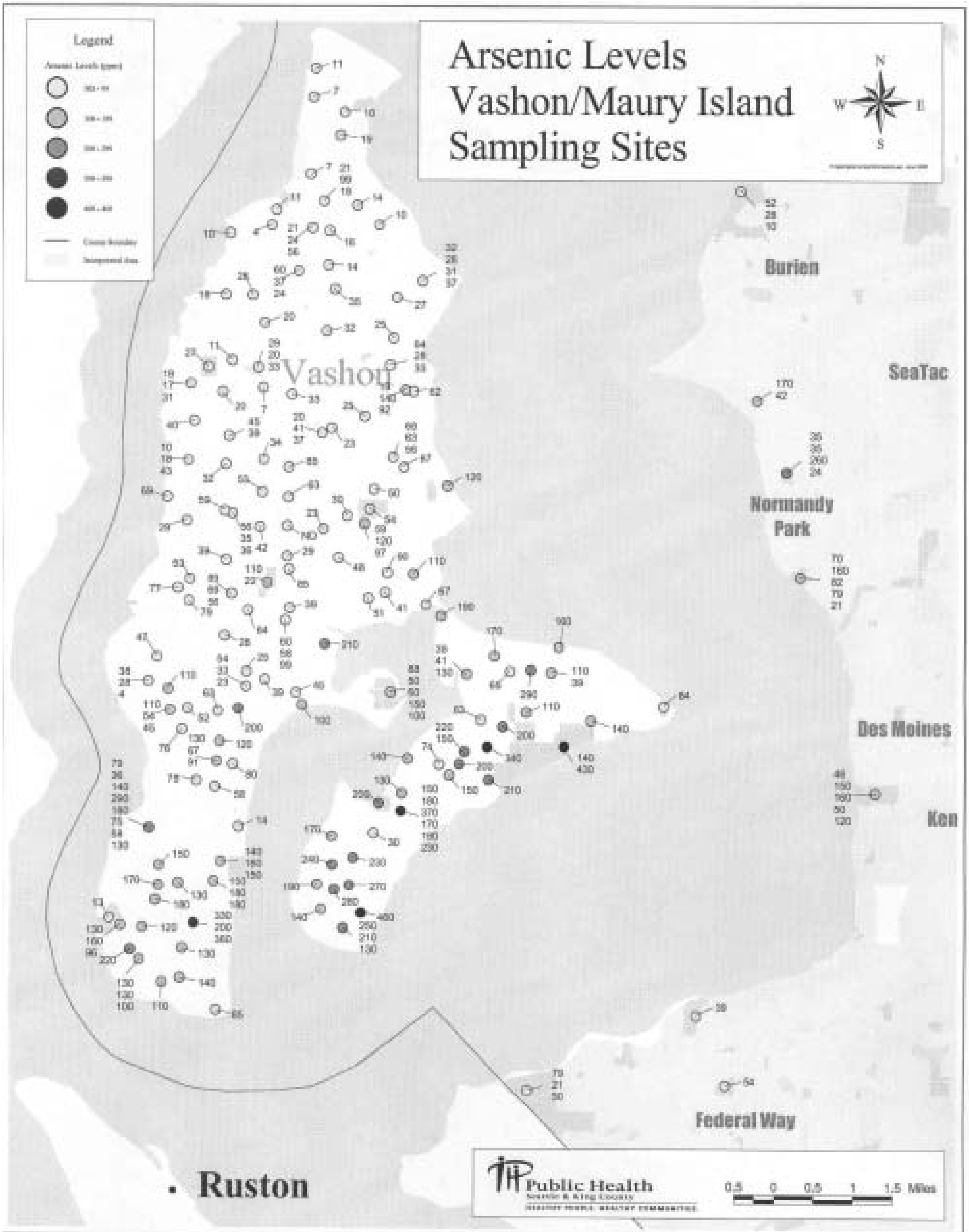

Arsenic Levels
Vashon/Maury Island
Sampling Sites
N
W
E
S
Legend
Burien
SeaTac
Vashon
Normandy
Park
Des Moines
Ken
Federal Way
Ruston
Public Health
Seattle & King County
0.5 0 0.5 1 1.5 Miles

Figure 7

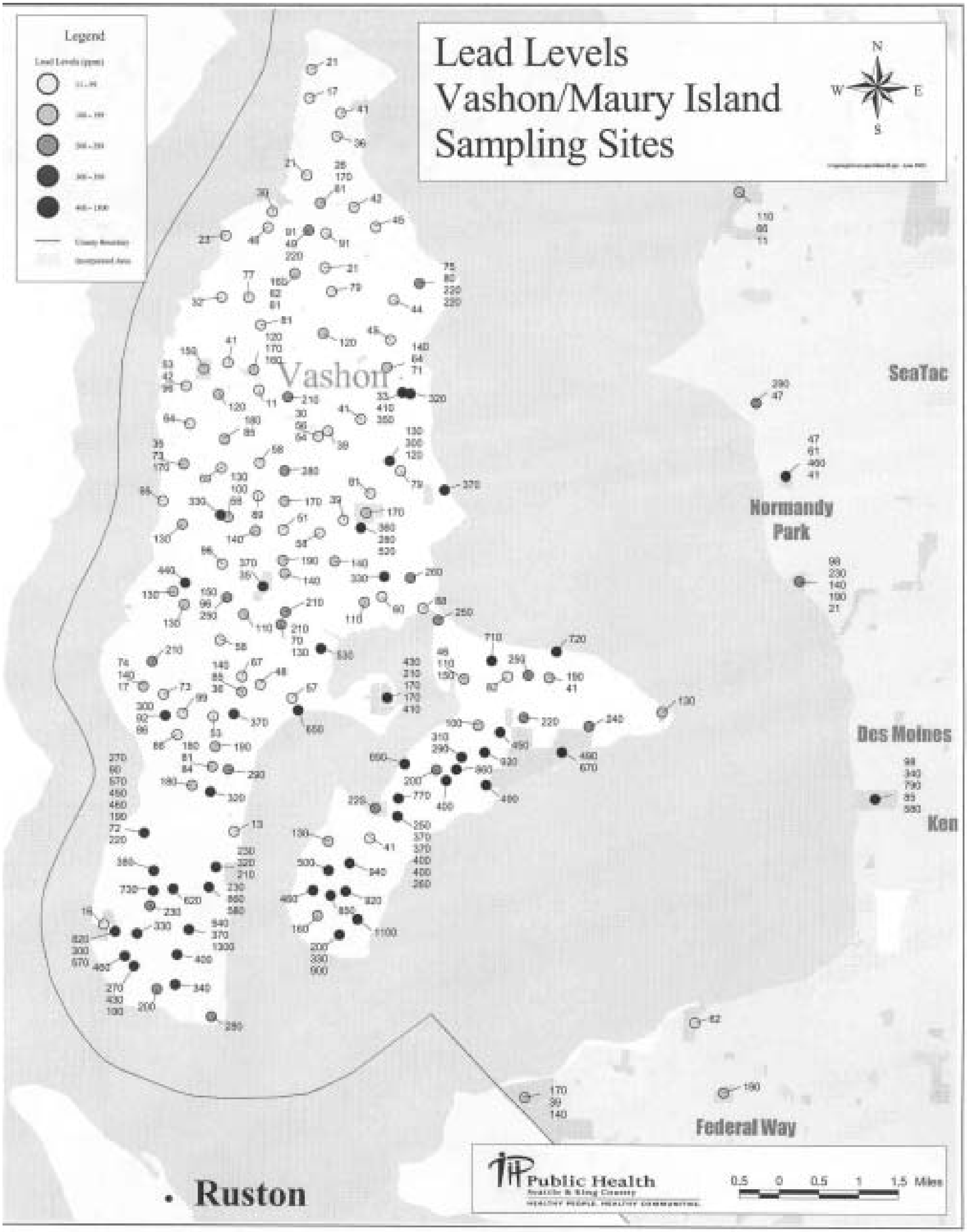

Lead Levels
Vashon/Maury Island
Sampling Sites
N
W
E
S
Legend
Vashon
SeaTac
Normandy Park
Des Moines
Federal Way
Ruston
Public Health
0.5 0 0.5 1 1.5 Miles

Figure 8

Multiple Box-and-Whisker Plots
Arsenic Data by Sampling Zone

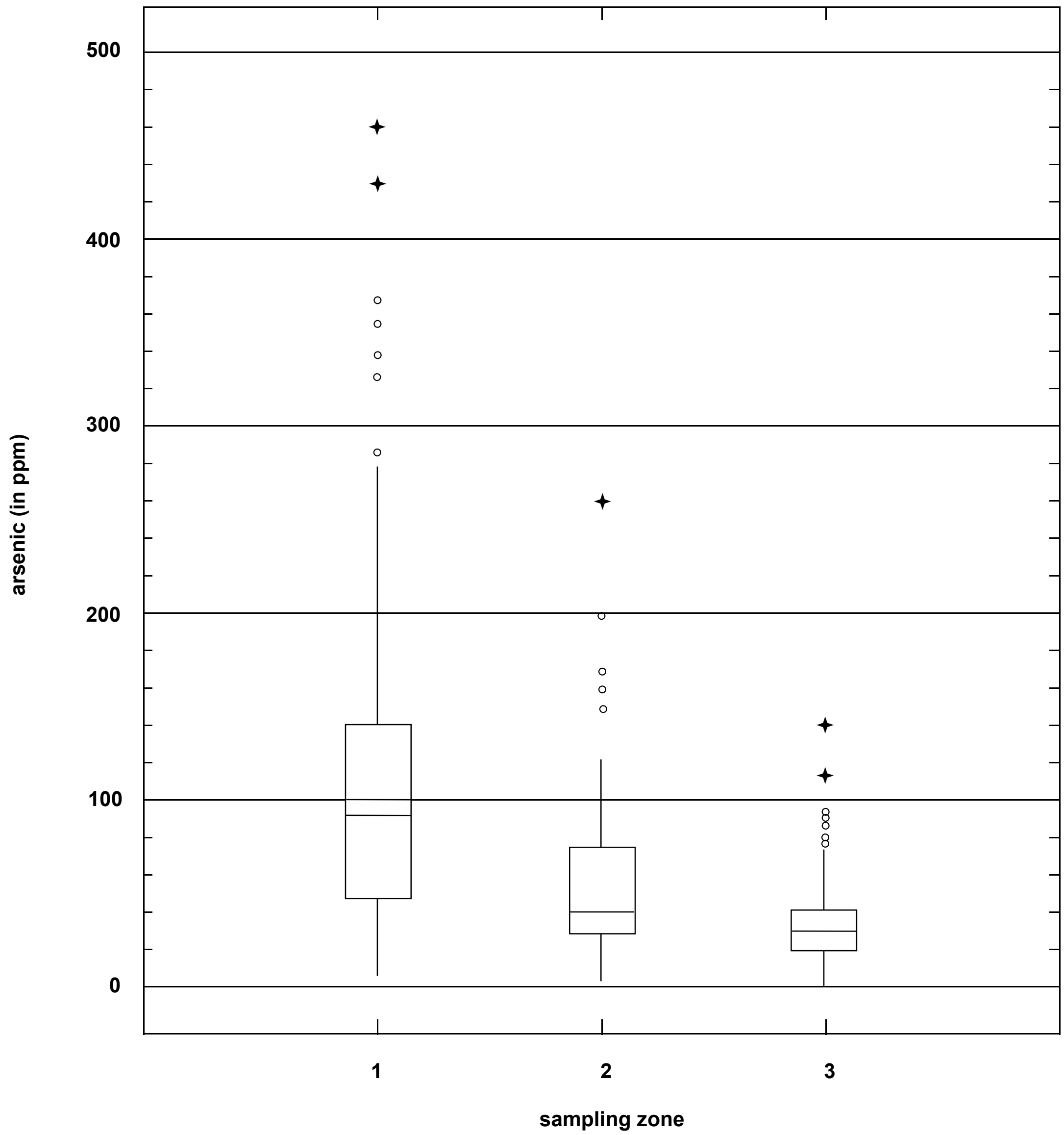


Figure 9

Multiple Box-and-Whisker Plots
Lead Data by Sampling Zone

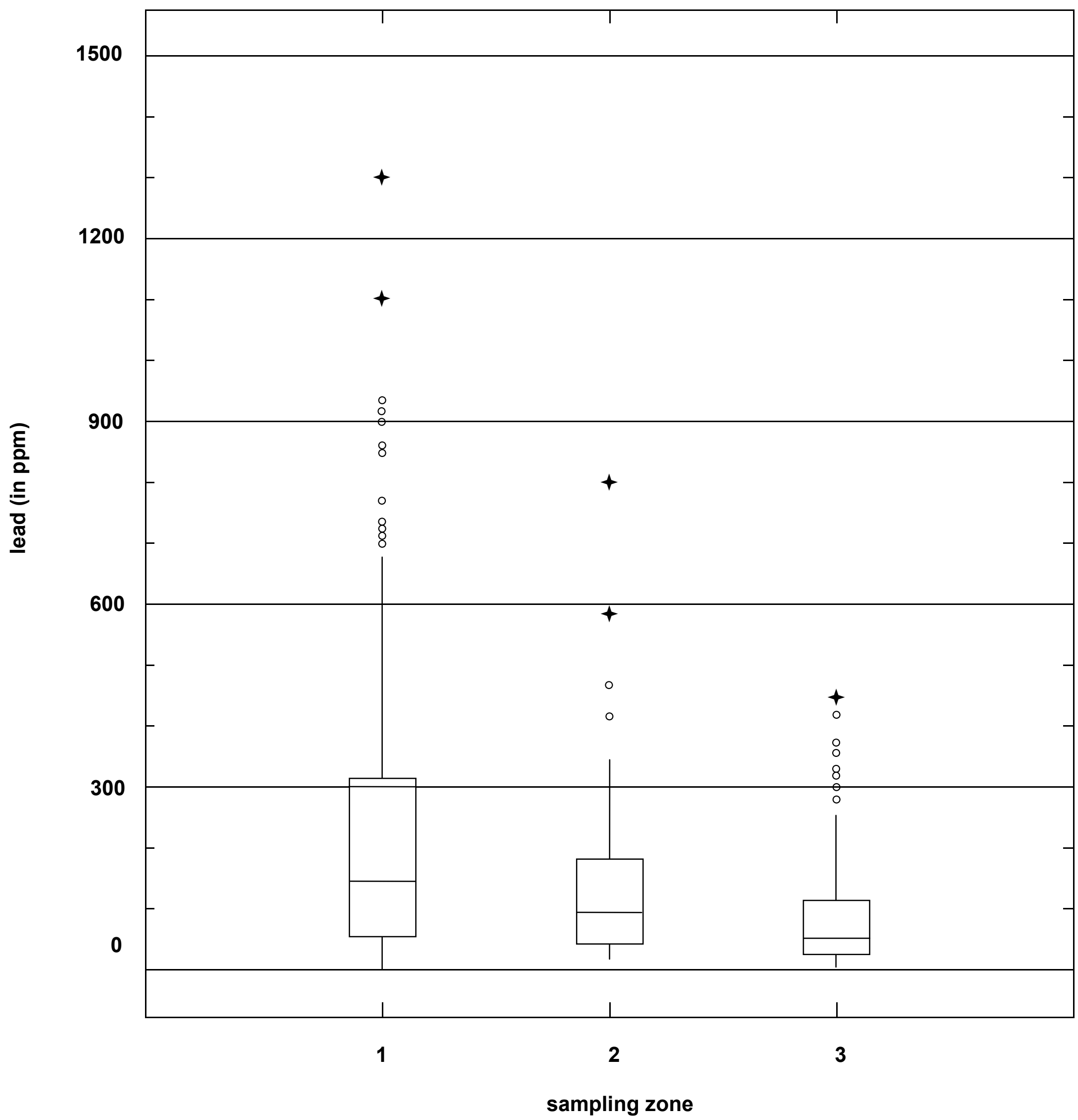


Figure 10

The Matrix

Reporting category Probability and Statistics

Overview Students place data in matrices and use mathematical conventions to locate particular data points

Related Standard of Learning 8.13

Objectives

- The student will describe the characteristics of a matrix, including designating labels for rows and columns.
- The student will use a matrix of no more than twelve entries to organize and describe a data set.
- The student will identify the position of an element by row and column.
- The student will transfer data from a chart to a matrix.

Instructional activity

1. Ask the students, “Have you ever used data that is in a spreadsheet or arranged data in a table? “This rectangular arrangement of data is very much like a matrix. Look at the data in the table below.

	Percent of Teenage Computer Users					
	Computer Games	**Music Programs**	**Educational Programs**	**Graphics Programs**	**Spread-Sheets**	**E-Mail**
Boys	80	43	43	47	30	29
Girls	74	32	51	54	36	32

Source: The Roper Organization, 1998 Roper Youth Report

2. Explain that if you do away with the labels and outlines, you have a rectangular arrangement of values. This is a *matrix*. Name the data shown above *Matrix D.*

$$D = \begin{bmatrix} 80 & 43 & 43 & 47 & 30 & 29 \\ 74 & 32 & 51 & 54 & 36 & 32 \end{bmatrix}$$

3. The *dimensions* of a matrix refer to the number of rows and columns in the matrix. Matrix D has 2 rows and 6 columns. It is a 2-by-6 (two-by-six) matrix.
4. Each value (called an *element*) in a matrix has a position within the matrix. The position of an element in the matrix is important. The element is located by the number of its row and the number of its column. For example, the element, 47, in matrix D, is in the first row and the fourth column. Its position is 1, 4.

**Table 1. Weighted Average Poverty Thresholds for Families of Specified Size
1959 to 1969**

	Unrelated individuals			2 people		
Calendar year	**All ages**	**Under age 65**	**Age 65 or older**	**All ages**	**Householder under age 65**	**Householder age 65 or older**
1959	$1,467	$1,503	$1,397	$1,894	$1,952	$1,761
1960	1,490	1,526	1,418	1,924	1,982	1,788
1961	1,506	1,545	1,433	1,942	2,005	1,808
1962	1,519	1,562	1,451	1,962	2,027	1,828
1963	1,539	1,581	1,470	1,988	2,052	1,850
1964	1,558	1,601	1,488	2,015	2,079	1,875
1965	1,582	1,626	1,512	2,048	2,114	1,906
1966	1,628	1,674	1,556	2,107	2,175	1,961
1967	1,675	1,722	1,600	2,168	2,238	2,017
1968	1,748	1,797	1,667	2,262	2,333	2,102
1969	1,840	1,893	1,757	2,383	2,458	2,215

Source: www.census.gov/income/histpov/hstpov1.lst

5. Convert the data in Table 1 into a matrix.
6. What are the dimensions of your matrix?
7. What position is $1,626 in? What element is in position 5,3?

Sample assessment

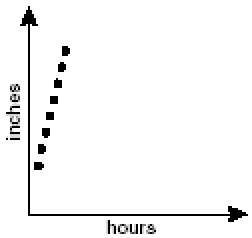
- While students are working, assess understanding.

Follow-up/extension

- Homework could consist of repeating the activity, using newspapers, menus, or simple interest.

Sample released test items

40 Which scatterplot best shows the relationship between a person's height and the time that person spends watching television?

F


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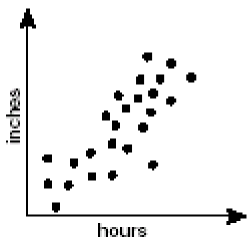

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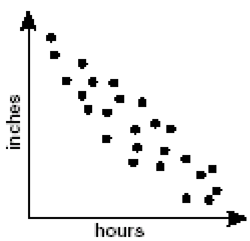

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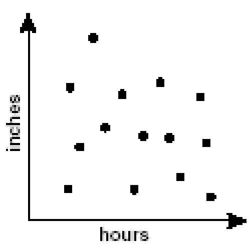
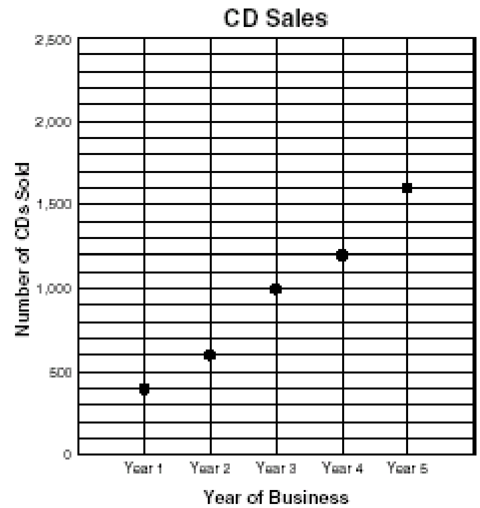


37 The graph shows the number of CDs sold each year at a small music store.



If the number of CDs sold each year continues to increase as shown in the plot, which is the *best* prediction of the number of CDs the store will sell during its 8th year of business?

A 3,400
B 3,000
C 2,400
D 2,000

Organizing Topic Patterns, Functions, and Algebra

Standards of Learning

8.14 The student will
a) describe and represent relations and functions, using tables, graphs, and rules; and
b) relate and compare tables, graphs, and rules as different forms of representation for relationships.

8.15 The student will solve two-step equations and inequalities in one variable, using concrete materials, pictorial representations, and paper and pencil.

8.16 The student will graph a linear equation in two variables, in the coordinate plane, using a table of ordered pairs.

8.17 The student will create and solve problems, using proportions, formulas, and functions.

8.18 The student will use the following algebraic terms appropriately: *domain, range*, *independent variable,* and *dependent variable*.

Essential understandings, knowledge, and skills	Correlation to textbooks and other instructional materials
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to	
• Substitute numbers for variables in an algebraic expression and simplify the expression by using the order of operations. Exponents used are whole numbers less than 4.	___
• Apply the order of operations to evaluate formulas.	___
• Solve two-step linear equations by showing the steps and using algebraic sentences.	___
• Solve two-step inequalities by showing the steps and using algebraic sentences.	___
• Substitute known values for variables in a formula.	___
• Solve a formula by using algebraic procedures.	___
• Construct a table of ordered pairs by substituting values for x in a linear equation to find values for y.	___
• Plot in the coordinate plane ordered pairs (x, y) from a table.	___
• Connect the ordered pairs to form a straight line.	___
• Graph in a coordinate plane ordered pairs that represent a relation.	___
• Write a rule that represents a relation from a table of values.	___
• Write a table of values from the rule that represents a relation.	___
• Write a table of values from the graph of ordered pairs of a relation.	___

- Describe and represent relations and functions, using tables, graphs, and rules. ____________________________
- Relate and compare different representations of the same relation. ____________________________
- Apply the following algebraic terms appropriately: *domain, range, independent variable,* and *dependent variable.* ____________________________
- Identify examples of domain, range, independent variable, and dependent variable. ____________________________
- Determine the domain of a function. ____________________________
- Determine the range of a function. ____________________________
- Determine the independent variable of a relationship. ____________________________
- Determine the dependent variable of a relationship. ____________________________

Algeblocks™ and Equation Solving

Reporting category	Patterns, Functions, and Algebra
Overview	Students develop the concepts associated with solving equations, using manipulatives.
Related Standard of Learning	8.15

Objective

- The student will solve two-step equations and inequalities in one variable, using concrete materials, pictorial representations, and paper and pencil.

Materials needed

- Algeblocks™ for each student or pair of students
- Handouts from Algeblocks™ Labs 10-2 through 10-8, one copy for each student

Instructional activity

1. Use the Algeblocks™, Teacher's Resource Binder, Chapter 10, Labs 10-2, 10-3, 10-4, 10-5, 10-6, 10-7, and 10-8. Distribute handouts.
2. Emphasize the connection among representations (concrete, pictorial, and symbolic), and ensure that students are secure in their understanding of all three representations.
3. Reinforce the use of the field properties of real numbers and the properties of equality in justifying steps in solving equations.
4. Have the students confirm solutions to equations with the graphing calculator and by graphing the solutions in the coordinate plane on paper.

Sample assessment

- Included in the Algeblocks™ labs

Follow-up/extension

- Algeblocks™ Lab 10-10

Homework

- "Think about It" from the Algeblocks™ labs

Algeblocks™: Solving Inequalities

Reporting category	Patterns, Functions, and Algebra
Overview	The lesson allows students to use Algeblocks™ to conceptualize solving inequalities.
Related Standard of Learning	8.15

Objective

- The student will solve linear inequalities in one variable and graph the solution in the coordinate plane.

Materials needed

- Algeblocks™ for each student or pair of students

Instructional activity

1. Use the Algeblocks™ Teacher's Resource Binder, Chapter 10, Lab 10-9, "Solving Inequalities."
2. Emphasize the connection among representations (concrete, pictorial, and symbolic), and ensure that students are secure in their understanding of all three representations.
3. Reinforce the use of the field properties of real numbers and the properties of order in justifying steps in solving inequalities.

Sample assessment

- Embedded in the Algeblocks™ lab

Homework

- "Think about It" from the Algeblocks™ labs

Cover-up Problems

Reporting category	Patterns, Functions, and Algebra
Overview	This is a quick, intuitive lesson for students to derive a method of solving equations in one variable.
Related Standard of Learning	8.15

Objective

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to solve two-step linear equations by showing the steps and using algebraic sentences.

Materials needed

- A “Cover-up Problems” handout for each student

Instructional activity

1. Work through the equations, leading the students to solve the equations for x.
2. Conversations about *how* the students know their answers and *what operations* they performed to get them are important. Have students use the properties of real numbers and equality to justify steps in equation solutions.
3. Identify and discuss inverse operations.

Sample assessment

- Choose another “covered” equation to see whether the students can work through the problem and explain their thinking.

Follow-up/extension

- Have the students record their steps with one-variable equations.

Cover-up Problems

Equation 1: $4 * x = 12$

What is the value "under the hand"?

 = 12

Now what is the value "under the hand"?

4 *  = 12

Equation 2: $x + 7 = 43$

What is the value "under the hand"?

 = 43

Now what is the value "under the hand"?

 + 7 = 43

Equation 3: $x - 11 = 15$

What is the value "under the hand"?

 = 15

Now what is the value "under the hand"?

 – 11 = 15

Equation 4: $\frac{x}{5} = 10$

What is the value "under the hand"?

 = 10

Now what is the value "under the hand"?


—— = 10
5

Equation 5: $4 * x + 8 = 12$

What is the value "under the hand"?

 = 12

Now what is the value "under the hand"?

 + 8 = 12

Now what is the value "under the hand"?

4 *  + 8 = 12

Equation 6: $\frac{x}{5} - 1 = 4$

What is the value "under the hand"?

 = 4

Now what is the value "under the hand"?

 – 1 = 4

Now what is the value "under the hand"?


—— – 1 = 4
5

Equation 7: $\frac{2 * x}{3} + 7 = 13$

What is the value "under the hand"?

 + 7 = 13

Now what is the value "under the hand"?


—— + 7 = 13
3

Now what is the value "under the hand"?

2 * 
—— + 7 = 13
3

Equations and Inequalities

(Lesson © 2000 by Math.com)

Reporting category Patterns, Functions, and Algebra

Overview Students solve equations.

Related Standard of Learning 8.15

Objective

- The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to solve two-step linear equations by showing the steps and using algebraic sentences.

Materials needed

- Computer lab with a computer for every student or a computer with a viewing system that may be seen by all the students in the class

Instructional activity

2. Have students access the Web site http://www.math.com/school/subject2/lessons/S2U1L2GL.html
3. Direct the students to work through the steps “First Glance,” “In Depth,” “Examples,” and “Workout.”

Sample assessment

- “Workout” from step 2 above.

Graphing Equations and Inequalities: The Coordinate Plane

(Lesson © 2000 by Math.com)

Reporting category	Patterns, Functions, and Algebra
Overview	Students graph in the coordinate plane.
Related Standards of Learning	8.14, 8.16

Objectives

- The student will construct a table of ordered pairs by substituting values for x in a linear equation to find values for y
- The student will plot in the coordinate plane ordered pairs (x, y) from a table
- The student will connect the ordered pairs to form a straight line
- The student will graph in the coordinate plane ordered pairs that represent a relation
- The student will write a rule that represents a relation from a table of values
- The student will write a table of values from the rule that represents a relation
- The student will write a table of values from the graph of ordered pairs of a relation
- The student will describe and represent relations and functions, using tables, graphs, and rules.
- The student will relate and compare different representations of the same relation.

Materials needed

- Computer lab with a computer for every student or a computer with a viewing system that may be seen by all the students in the class

Instructional activity

1. Have students access the Web site http://www.math.com/school/subject2/lessons/S2U4L1GL.html and work through the steps "First Glance," "In Depth," "Examples," and "Workout."
2. Provide other opportunities for students to use linear functions to generate tables of values, graph the ordered pairs, and draw the line.

Sample assessment

- "Workout" from step 1 above.

Independent and Dependent Variables

(This lesson derived from Math Connects: Patterns, Functions, and Algebra*)*

Reporting category	Patterns, Functions, and Algebra
Overview	Students differentiate between dependent and independent variables.
Related Standard of Learning	8.18

Objectives

- The student will apply the following algebraic terms appropriately: *domain, range, independent variable,* and *dependent variable*
- The student will identify examples of domain, range, independent variable, and dependent variable
- The student will determine the independent variable of a relationship
- The student will determine the dependent variable of a relationship.

Materials needed

- Three pendulums of different lengths (38 cm, longer than 38 cm, shorter than 38 cm) with a penny taped to the swinging end of each for weight

Instructional activity

Part I

1. Allow the students to watch the video from Math Connects: "Patterns, Functions, and Algebra."
2. Ask the students, "Have you ever had a cuckoo clock or grandfather clock? Did you watch the mechanism? Did you ever have to make an adjustment to speed up or slow down the time?" The activity in this class will help students understand the pendulum action of the clock mechanism while they explore data in a variety of graph forms. In addition, the concept of *variable* will be extended, and the relationship between dependent and independent variables will be explored.
3. The basis of the activity and graphs will establish the occurrence of algebra concepts in everyday life.
4. Have students conduct an experiment with their swingers (pendulums), following these steps:
   a. Tape a pencil to the edge of a desk or table.
   b. Place the 38 cm swinger on the pencil (loop it over the pencil).
   c. Hold the pendulum parallel to the table. Release it and count the number of swings (a *swing* is defined as a complete back and forth movement) that occur during 15 seconds. Record your results.
   d. Hold the 38 cm swinger at a 45-degree angle from the edge of the table. Conduct the experiment again. Record your results.
   e. Place a second penny on the end of the 38 cm swinger, and repeat the experiment as in step c.
   f. With your longer swinger, follow the directions in steps c, d, and e.
   g. With your shorter swinger, follow the directions in steps c, d, and e.

Questions for reflection

- Why was it important to add a penny and to change the angle of the swinger in our experiment?

- How are graphs useful as information sources?

Part II

1. Tell the students that the purpose of part II is to prepare pendulums to determine which variables, if any, affect their behavior.
2. After students make swingers, have each tape a pencil to the edge of the desk.
3. Ask students to predict how many times they think the swinger will swing in 15 seconds.
4. Have a mock start: say “ready, set . . .”
5. Wait for students to ask two questions: How high? (even with desk edge) What constitutes a cycle? (back and return)
6. Answer these questions, then allow them to swing while you keep time.
7. Students should get 12 swings, but an occasional 11 or 13 is okay.
8. Ask students what they could change in their swinger system that would affect the number of swings. (Students should suggest weight, release position, and/or length.)
9. Introduce concept of *variable* — anything that can be changed that might affect the overall outcome of the experiment.
10. Write *variable* on board and define.
11. Review setup for the pendulum: introduce the “standard pendulum system.” The standard here is 38 cm string, one penny, parallel to table, 15 seconds.
12. Introduce the idea of an experiment — an experiment is an investigation designed to see how a variable affects the outcome of an event. A controlled experiment is one in which one variable is changed and the outcome is compared to the standard.
13. Have students perform the two experiments to test the variables: (a) weight: add a second penny and swing for 15 seconds. (b) release position: release at a 45-degree angle from desk edge. Before each experiment, have students predict what they think will happen. Students should again get 12 swings in each experiment.
14. Hang pendulums on swinger number line. (All on number 12.)
15. Have students make new swingers, each with varying lengths.
16. Have students swing the pendulums for 15 seconds, then hang on number line.
17. Draw a chalk line curve under the swingers on the number line. Point out that this is a concrete graph.
18. Make a picture graph — next level of abstraction. This is the representational level of abstraction. Have students draw the swingers exactly as they see them on the board.
19. Make a two-coordinate graph — the next level of abstraction, known as *symbolic*. Nothing looks or feels like the real swingers — everything is resolved to numbers. Students plot points representing the length of each swinger and the number of times it swung in 15 seconds. Have them place independent variables — what they know before the experiment — on the x-axis and place dependent variables — what they find out — on the y-axis.

Follow-up/extension

- *Algebra, Data, and Probability Explorations for the Middle School,* Dale Seymour Publications
  Activity 1-1 Bouncing a Tennis Ball Variables and Patterns
  Activity 1-4 Packing the Tennis Balls Variables and Patterns

- NCTM Addenda Series Patterns and Functions "Information from Graphs" p. 58-60.
  NCTM Addenda Series Developing Number Sense Activity 43 p. 47.
  NCTM Curriculum and Evaluation Standards, p. 101 (Figure 8.4)
- As a classroom activity, ask students to write about what one or more of these graphs might show. Then, students could take all the descriptions and, without looking at the graphs, draw a graph they think represents the description and use the vocabulary in this lesson on dependent and independent variables. Students would indeed be communicating and connecting while they are using logical reasoning.

Sample assessment

- Fill a cup with M&M's®. Count them to determine the initial sample size (*n*). This will be the value for t = 0.
- Shake the cup and pour out the M&M's® on the paper. Remove all the M&M's® with an "M" showing. Count the remaining M&M's®. This will be the value of *n* at t = 1.
- Put the remaining M&M's® in the cup, shake and pour out on the paper. Remove all M&M's® with an "M" showing. Count the remaining M&M's®. This is the value of *n* at t = 2.
- Repeat the process until there are no M&M's® with an "M" showing. You may now eat your experiment.
- Take the values from the table and graph them.

Table of Values for M&M's® Experiment

t	n
0	
1	
2	
3	
4	
5	
6	

- Explain in your own words what *t* and *n* represent.
- What generalizations can you make from the chart?
- What relationship do you notice to the swingers in the experiment?

Follow-up/extension

- Explain in your own words the difference between an *independent variable* and a *dependent variable.* Use an example and/or draw a picture if it helps you to communicate your thinking.
- When would a manual approach to solving a problem be more appropriate than using technology? Is one more useful than the other? When would using technology be more useful? Give an example(s).

The Scoop on Ice Cream

Reporting category Computation and Estimation/Patterns, Functions, and Algebra

Overview Students use data involving favorite ice cream flavors to predict the amount of each flavor that would be needed to serve a crowd of people.

Related Standards of Learning 8.3, 8.17

Objectives

- The student will use data to predict an outcome, using proportional reasoning.
- The student will use unit proportions and equivalencies for finding solutions.

Materials needed

- "Scoop-on-Ice-Cream Recording Sheet," one copy for each student
- Calculators

Instructional activity

1. *Initiating Activity:* Give each student a copy of the handout, "Scoop-on-Ice-Cream Recording Sheet," and discuss the information about favorite ice cream flavors. Ask, "What percent of the people prefer chocolate ice cream? Vanilla ice cream? Strawberry?"
2. Review with the class the important measurement equivalents that will help them solve the problem.
3. Read the Scrumptious Scoops problem (on the handout) with the class.
4. Have the class work in pairs to solve the problems concerning amounts of ice cream.
5. *Closing Activity:* When students have completed the problems, have them share their solutions and strategies for solving.

Sample assessment

- Pay particular attention to the conversions of units and the proportions used to find answers. Make sure students recognize amounts that "make sense" when their answers are calculated.

Solution

1. How many people will prefer chocolate ice cream?
   a. If 29% of the people prefer chocolate, then 29% of 650 people = (.29)(650) = 188.5 or 189. Hence, 189 people prefer chocolate.
   or
   b. If 29% of the people prefer chocolate, then $\frac{29}{100} = \frac{x}{650}$; therefore, $x = \frac{(650)(29)}{100}$ = 188.5 or 189. Hence, 189 people prefer chocolate.
2. How many half-gallons of chocolate ice cream will be needed?
   a. If $\frac{1}{2}$ cup is needed for each person, then $\frac{.5 \text{ cups}}{1 \text{ person}} = \frac{x \text{ cups}}{189 \text{ people}}$; therefore, $x = (189)(.5) =$ 94.5. Hence, 94.5 cups of chocolate ice cream are needed for 189 people.

b. If one half-gallon equals 8 cups, then $\frac{\text{1 half-gallon}}{\text{8 cups}} = \frac{x\text{ half-gallons}}{\text{94.5 cups}}$; therefore, $x = \frac{94.5}{8} =$ 11.81 or 12. Hence, 12 half-gallons of chocolate will be needed.

3. If everyone is served a scoop, how many half-gallons of ice cream will be served? How many pounds will that be?

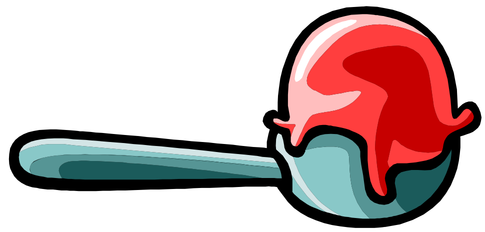
   a. If one scoop of ice cream equals $\frac{1}{2}$ cup, then the number of cups of ice cream needed for 650 people is (.5)(650) = 325 cups. If one half-gallon equals 8 cups, then the number of half-gallons of ice cream needed for 650 people is 325 ÷ 8 = 40.6 or 41 half-gallons of ice cream.

   b. If one gallon weighs 5 pounds, then one half-gallon weighs 5 ÷ 2 = 2.5 pounds. Therefore, 40.6 half-gallons weigh 101.5 or 102 pounds of ice cream.

Follow-up/extension

- Do the "The Ice Cream Recipe" activity found on the handout, either as an individual activity or as a whole-class activity.

Scoop-on-Ice-Cream Recording Sheet



The Three Favorite Flavors of Ice Cream*

Favorite Flavor	Percent of Those Polled
Vanilla	55%
Chocolate	29%
Strawberry	16%

Scrumptious Scoops is a very popular ice cream parlor in a small town in Virginia. To celebrate the Fourth of July, the store decided to serve free single scoops of its three most popular flavors to the audience at the Independence Day outdoor band concert. Mr. Scrumptious decided that he could determine how much ice cream he would need by using the data provided by the International Ice Cream Association. The town estimated that approximately 650 people would attend the band concert.

1. Assuming everyone will want a free scoop of ice cream, how many people would you expect to prefer chocolate?
2. How many half-gallons of chocolate ice cream should Mr. Scrumptious plan to have on hand to give to those people?
3. If the representatives from Scrumptious Scoops serve everyone at the band concert a scoop of ice cream, how many half-gallons of ice cream will they serve? How many pounds will that be?

Important Measurement Equivalents

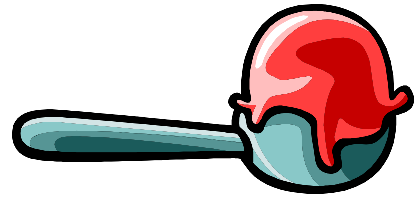
A gallon of ice cream weighs about 5 pounds and contains 4 quarts.

One scoop of ice cream is $\frac{1}{2}$ *cup or about 68 grams.*

One gallon contains 16 cups, so one half-gallon contains 8 cups.

*International Ice Cream Association Data

Ice Cream Recipe



Make ice cream in plastic zip-lock bags as follows:

1. Combine 3 tablespoons of sugar, a few drops of vanilla extract, and 1 cup of milk in a 1-quart zip-lock bag, and seal the bag tightly. You may add cookie pieces or well-drained fruit to your ice cream mixture if you wish.
2. Put about 2 cups of ice and $\frac{1}{2}$ cup of rock salt in a 1-gallon zip-lock bag. Use small ice cubes, or break the ice into small pieces.
3. Put the smaller bag into the larger bag, and seal it tightly. Then shake the large bag until the ice cream mixture freezes. This step takes some time.

Why do you think this method works for making ice cream?

____

____

____

Using the recipe above, how many students could make ice cream using 8 cups of sugar and 4 gallons of milk? To solve the problem you need to know: 1 cup = 16 tbsp. and 1 gallon = 16 cups. Show your calculations in the box below:

Would there be any ingredients left over? If so, how much? Show your thinking below:

Animals Count

Reporting category	Computation and Estimation/Patterns, Functions, and Algebra
Overview	Students simulate a method employed by biologists to estimate animal populations, using beans to represent the animals. The method, called "capture-tag-recapture," uses red and white beans to represent tagged and untagged populations of deer. During the simulation, proportions are developed for use in determining the total number of the entire population.
Related Standards of Learning	8.3, 8.17

Objective

- The student will use previously learned techniques for reasoning about ratios and proportions to estimate the size of a population when they cannot count the entire population.

Materials needed

- Small jar
- Large jar or other container with lid for each group of students
- Enough white beans to fill the jar of each group
- 100 red (kidney) beans for each group
- Paper cups or scoops for scooping beans
- "Animals-Count Recording Sheet," one copy for each group

Note: Two types (colors) of Goldfish™ crackers or other equal-sized objects can be substituted for beans.

Instructional activity

1. *Initiating Activity:* Discuss with the class: "How do wildlife officials determine how many sea otters there are in the United States? How do environmentalists decide whether an animal or plant should be on the endangered species list?" Answers will vary. Discuss scientific methodology for performing accurate counts of wildlife.
2. Demonstrate the experiment, using a small jar that is three-fourths full of white beans. Remove 10 beans and replace them with red beans. Cover the jar, shake, and remove a sample handful of beans.
3. Ask students to count the number of red beans and white beans in the sample. How can they set up a meaningful ratio using these numbers?

$$\text{Ratio} = \frac{\text{number of red beans in sample}}{\text{total number of beans in sample}}$$

Be sure the students recognize that at this point they know a) the number of marked (red) beans in the whole population and b) the number of marked (red) beans in the sample.

Ask the students to use what they know about making comparisons with ratios to estimate the total number of beans in the jar. Record their proportion and their estimate. Students should be able to record a proportion that shows the total number of red beans drawn in four trial samples related to the total number of all beans drawn in the four samples. This ratio is equal to the ratio of the total number of red beans (100) to the total population, which is the number being sought, x.

$$\text{Proportion: } = \frac{\text{total number of red beans (100)}}{\text{total population } (x)}$$

4. Have a group count the number of beans in the jar to see how close the estimate was to the actual number of beans.
5. Discuss with students the methodology shown by the experiment, which models what really happens when the capture-tag-recapture method is used to estimate the number of deer in a large area. Some factors that must be considered are tagging deer from several places in the area under study, taking a sufficient sample for tagging, allowing the tagged animals time to mix thoroughly with the population, and taking the final samples from several places in the area.
6. *Simulation Activity:* Provide each group of three or four students with a jar with a lid, a large number of white beans, and an “Animals-Count Recording Sheet.” Model the first trial of the deer-population experiment with the students, making sure that they know how to record their results.
7. Ask the groups of students, “How are you mixing the beans? How are you keeping track of what you know? How might the number of samples you take affect your estimate?”
8. Have each group remove 100 white beans from the jar and set them aside.
9. Have them place 100 red beans representing the tagged deer into the jar to replace the white beans removed.
10. Have them shake the jar to mix the beans and then scoop out a cupful of beans without looking at them.
11. Have them record on the recording sheet the number of red beans and the total number of beans in the sample.
12. Have them repeat this scoop-and-count procedure three more times. In each case, have them record on the recording sheet the number of red beans and the total number of beans in the sample.
13. *Closing Activity:* Have the groups study the data they collected and use the data to estimate the number of beans in the jar. Have each group report, explaining how they made their estimate and showing any calculations they made. The proportions should use the reasoning about the tagged deer and the total population as shown in the sample (see Initiating Activity, steps #3 and 4).

Sample assessment

- As student groups are working, circulate in the room to make sure they are accurately recording all needed information and to assess their understanding of ratios. As groups report, make sure their proportions make sense mathematically. Students should be able to articulate their answers and how they relate to the proportionality of the capture-tag-recapture method.

Follow-up/extension

Additional problems using this method could be assigned for students struggling with forming their proportions. For instance, similar real-life problems might involve fish in a lake or beavers in a wooded area.

Snowy Egrets

Reporting category	Computation and Estimation/Patterns, Functions, and Algebra
Overview	Students use the results of a capture-tag-recapture experiment along with proportional reasoning to determine the number of snowy egrets in a population. There is no simulation for this experiment; students will apply the methodology of the deer simulation used in the "Animals Count" activity.
Related Standards of Learning	8.3, 8.17

Objective

- The student will use techniques he or she has learned for reasoning about ratios and proportions to estimate the size of a population when he or she cannot count the entire population. In this problem, there will be a direct application of proportional reasoning, using available data.

Materials needed

- "Snowy Egrets Recording Sheet," one copy for each student

Instructional activity

1. *Initiating Activity:* Distribute the "Snowy Egrets Recording Sheet" and review the background information with the class. Answer questions as they arise.
2. Ask the students to use the methodology from the deer capture-tag-recapture experiment in the "Animals Count" activity to solve the problem. Make sure they show the proportion they use and explain the strategy for their estimate.

$$\frac{2 \text{ tagged egrets}}{50 \text{ egrets}} = \frac{20 \text{ tagged egrets}}{x \text{ egrets}}$$

3. *Closing Activity:* Discuss the estimates with the class. Ask for volunteers to explain why they are confident that their answer is correct. Discuss with the class *why* proportional reasoning works for this kind of problem.

Sample assessment

- Students should be able use the data from the problem to form the proportion. One method of solving this proportion is to solve the equation for x.

$$x = \frac{(20)(50)}{2} = 500 \text{ egrets}$$

- Another method is to realize that 20 is 10 times 2, therefore x would be 10 times 50, or 500 egrets.

Follow-up/extension

- Additional problems using this method could be assigned for students struggling with forming their proportions. For instance, a problem involving fish in a lake or one involving beavers in a wooded area would be good examples of real-world situations that require similar reasoning.

Snowy Egrets Recording Sheet

The U.S. Department of Natural Resources manages the bird population in the Chesapeake Bay area on the eastern shore of Maryland. The Department has been tracking the snowy egret population and has discovered that snowy egrets tend to migrate to the same geographical area each year during the warmer spring and summer months. The Department is worried that urban sprawl may be affecting the egret population adversely.

For the past several summers, a biologist studied the snowy egret, a white shore bird that frequents the eastern shore of Maryland near St. Michaels during the summers. Two summers ago, she trapped 20 snowy egrets, tagged, and released them. This past summer, she trapped 50 snowy egrets and found that two of them were tagged. She used this information to estimate the total snowy egret population on the eastern shore of Maryland.

1. Using the biologist's findings, estimate the number of snowy egrets on the eastern shore of Maryland. Explain how you made your estimate.

2. How confident are you that your estimate is accurate? Explain your answer.

Population Density

Reporting categories Computation and Estimation/Patterns, Functions, and Algebra

Overview Students determine a method of calculating population density, using themselves and a designated 2-yd.-by-2-yd. square. Population density will be represented in two ways.

Related Standards of Learning 8.3, 8.17

Objectives

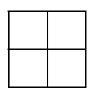
- The student will create a unit ratio.
- The student will use proportional reasoning in a practical application.

Materials needed

- Masking tape
- Measuring tape
- Calculators

Instructional activity

1. *Initiating Activity:* To help the class grasp the concept of population density, conduct the following activity. Have the students use the measuring tape to mark off on the floor a square that is approximately 2 yards by 2 yards. Outline the outside square and the grid with masking tape.



2. Have eight students stand within the 2 -yard-by-2-yard square. Discuss as a group possible methods of determining the number of students per square yard. The discussion should move to a "unit ratio" of two people in one square yard. Be certain that the class understands the term unit ratio.
3. Ask, What is the "mean number" of people per square yard? Make sure the students understand the meaning of the term *mean* and that the mean number indicates the *average* number. Show them that in this case, it is another way of representing the unit ratio, or two people per one square yard.
4. Have the eight students reposition themselves a number of times. Each time ask what is the mean number of people per square yard. The class should come to understand that no matter how the eight students are arranged within the space, there will always be two students per one square yard and that this ratio represents the population density for the taped area.
5. *Closing Activity:* Extend the problem to 15 people. What is now the mean number of people in one square yard? Be sure that the class understands why there are 3.75 people per one square yard.

Sample assessment

- Make sure the concepts of *mean* and *unit ratio* are understood. Check student answers for the problem of 10 people in the square to be sure the concept application was extended to a different number.

Virginia Population Density

Reporting category Computation and Estimation/Patterns, Functions, and Algebra

Overview Given the population numbers of Virginia and West Virginia and the number of square miles of land in each state, students will determine population density for each state and then decide which state has the greater population density. Based on this information, students will develop a strategy that will help them determine how many citizens of one state would have to move to the other state to make the population densities equal.

Related Standards of Learning 8.3, 8.17

Objective

- The student will work with population densities and unit proportions to understand the relationship of population to available area as a population density.

Materials needed

- Calculators
- "Virginia Population Density Recording Sheet," one copy for each student

Note: It is helpful for the students to first complete the "Population Density" activity in order to understand the concept of unit proportions. Refer back to the concept of mean or average population.

Instructional activity

1. *Initiating Activity:* Give the students copies of the "Virginia Population Density Recording Sheet" and the following data taken from the 1990 Census:

	Virginia	West Virginia
Population	6,187,000 people	1,793,000 people
Land Area	40,767 sq. mi.	24,231 sq. mi.

2. Ask the students to develop a strategy to determine the population density of each state and then calculate the population densities.
3. Instruct the students to determine which state has the greater population density and then to determine how many citizens of one state would have to move to the other state to create equal *population densities* (not equal *populations*). Remind the students that they should be prepared to describe the strategy used and to explain their reasoning.
4. *Closing Activity:* When the students are finished, have them explain their strategies for solution to the class. Their solutions should include information about how many citizens would have to move from one state to the other. Answers should include all calculations.

Sample assessment

- Monitor the class as the students work on and develop the following solutions. If incorrect strategies are conceived, note where the students lost the right track and lead them back to the proper process by asking questions.

Solutions

- To determine the population densities of the two states, make sure the students set up the proper ratios and solve them correctly:
  - Virginia: $\frac{6{,}187{,}000 \text{ people}}{40{,}767 \text{ sq. mi.}} = \frac{x \text{ people}}{1 \text{ sq. mi.}}$; therefore, $x = 151.76$ people per square mile, which is the population density of Virginia.
  - West Virginia: $\frac{1{,}793{,}000 \text{ people}}{24{,}231 \text{ sq. mi.}} = \frac{x \text{ people}}{1 \text{ sq. mi.}}$; therefore, $x = 74.00$ people per square mile, which is the population density of West Virginia.

  Now the students can see that Virginia has a higher population density than West Virginia.

- An alternative method of representation is the following:
  - Virginia: $\frac{40{,}767 \text{ sq. mi.}}{6{,}187{,}000 \text{ people}} = .007$ square miles per person in Virginia
  - West Virginia: $\frac{24{,}231 \text{ sq. mi.}}{1{,}793{,}000 \text{ people}} = .013$ square miles per person in West Virginia

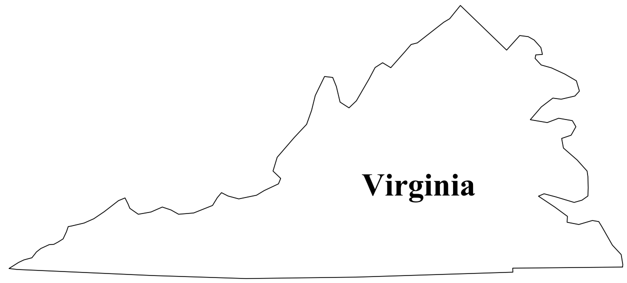
  While this method does not yield population density, it does show that with less area of land per person, Virginia is more densely populated than West Virginia.

- It is plain that Virginia citizens would have to move to West Virginia to equalize the population densities. There is more than one way to find out how many people would have to move. A possible solution using proportional reasoning is the following:
  - Find the total area in both states. (64,998 square miles)
  - Find the total population of both states. (7,980,000 people)
  - Find the ratio of the total population of both states to the total area of both states; this ratio represents the mean or average population density that needs to be achieved if the population densities are to be equalized. ($\frac{7{,}980{,}000 \text{ people}}{64{,}998 \text{ sq. mi.}} = \frac{x \text{ people}}{1 \text{ sq. mi.}}$; therefore, $x = 122.77$ people per square mile)
  - Subtract this average population density from the population density of Virginia. ($151.76 - 122.77 = 28.99$ people per square mile) This tells the number of Virginia citizens per square mile who would have to move to West Virginia.
  - Multiply this number of Virginia citizens per square mile who would have to move times the total number of square miles in Virginia ($28.99 \times 40{,}767 = 1{,}181{,}835$ people). This is the number of Virginia citizens who would have to move to West Virginia to equalize the population densities.

Follow-up/extension

- Have the students confirm or prove their solutions to the problem of how many Virginia citizens would have to move. Have them subtract this number of people (1,181,835 people) from the total population of Virginia and then add this same number of people to the total population of West Virginia, thereby creating the census after the move takes place. Then have them use these new population numbers to refigure the population densities of both states. They should quickly discover that the new population densities are the same — 122.77 people per square mile.

Virginia Population Density



Population: 1,793,000 people
Area: 24,231 sq. mi.

Population: 6,187,000 people
Area: 40,767 sq. mi.

1. Which state, West Virginia or Virginia, has the greater population density? Show your work.

2. How many citizens of one state would have to move to the other state to create equal *population densities*? Be prepared to tell the strategy you used and explain your reasoning.

Gridlock

Reporting categories	Patterns, Functions, and Algebra
Overview	Students use proportional reasoning to apply their experience with unit ratio problems about population density to a new situation. They do this in order to create another example of unit ratio and use that ratio to create a proportional reasoning situation.
Related Standard of Learning	8.17

Objective

- The student will work with unit ratios and proportional reasoning to solve a problem.

Materials needed

- "Gridlock Recording Sheet," one copy for each student
- Calculators

Note: This activity is designed to follow the unit ratio problems about population density found in the "Population Density" and "Virginia Population Density" activities. A brief review/discussion of unit ratios may be helpful before beginning the investigation.

Instructional activity

1. *Initiating Activity:* Distribute the "Gridlock Recording Sheet," and pose the following problem: "According to the *Guinness Book of Records*, Hong Kong is reported to have the highest traffic density in the world. In 1992, there were <u>418 registered cars and trucks per mile of road</u>. Another way to represent this ratio would be to say there are about <u>12.63 feet of road per registered vehicle</u> in Hong Kong."
2. Ask how the *Guinness Book of Records* determined these two different unit ratios.
3. Ask what the units are in each statement.
4. *Closing Activity:* Present the students with the following situation: "The city of Beetleville has 450,237 registered vehicles per 3,000 miles of road. What is the traffic density of Beetleville? Be prepared to explain your solution."
5. Ask the students to calculate both the number of vehicles per mile of road and the number of feet of road per vehicle.

Sample assessment

- Be certain that the students understand the solutions to the problems, as follows:
  - **Hong Kong problem:** If there are 418 vehicles per mile of road, then the ratio is $\frac{418 \text{ cars}}{1 \text{ road mile}}$.

    If I want to convert to feet, I know that 1 mile = 5,280 ft. Therefore, $\frac{418 \text{ cars}}{5{,}280 \text{ road ft.}} = \frac{1 \text{ car}}{x \text{ road ft.}}$

    . So $x = \frac{5{,}280}{418}$ = 12.63 road feet per vehicle.

- **Beetleville problem:** The city of Beetleville has $\frac{450{,}237 \text{ vehicles}}{3{,}000 \text{ road miles}} = 150.1$ vehicles per road mile. Therefore, $\frac{150.1 \text{ vehicles}}{5{,}280 \text{ road ft.}} = \frac{1 \text{ vehicle}}{x \text{ road ft.}}$. So, $x = \frac{5{,}280}{150.1} = 35.18$ road feet per vehicle.

Gridlock Recording Sheet

1. According to the *Guinness Book of Records,* Hong Kong is reported to have the highest traffic density in the world. In 1992, there were 418 registered cars and trucks per mile of road. Another way to represent this ratio would be to say there are about 12.63 feet of road per registered vehicle in Hong Kong. How did the *Guinness Book of Records* determine these two different unit ratios? What are the units in each statement? (For this problem, remember that 1 mile = 5,280 ft.)

2. The city of Beetleville has 450,237 registered vehicles for 3,000 miles of road. What is the traffic density of Beetleville? Calculate both the number of vehicles per mile of road and the number of feet of road per vehicle. Be prepared to explain your method of calculation.

Waste Paper

Reporting category	Computation and Estimation/Patterns, Functions, and Algebra
Overview	Students use proportional reasoning to determine the number of cubic yards of landfill space that can be saved by recycling the Sunday newspapers.
Related Standards of Learning	8.3, 8.17

Objective

- The student will use unit ratios and formulate proportions to solve contextual problems about recycling and environmental issues.

Materials needed

- "Waste Paper Recording Sheet," one copy for each student
- Calculators

Note: Reviewing volume concepts (cubic measurements) with the students might help them solve this problem more efficiently.

Instructional activity

1. *Initiating Activity:* Hand out a "Waste Paper Recording Sheet" to each student, and read the recycling example #1 with the class.
2. Have students work in pairs to develop a strategy to solve the problem. Once they complete their work, have them share their strategies for solution and compare their answers.
3. *Closing Activity:* Continue to example #2 and read it with the class. Have the students solve the problem and explain their strategies.

Sample assessment

- Monitor the class as the pairs of students work on and develop the following solutions. If incorrect strategies are conceived, note where the students lost the right track and lead them back to the proper process by asking questions.
- Solutions
  - Example #1: If 100% of the newspapers were recycled, then producing the Sunday papers with recycled paper would save 500,000 trees. Dividing 500,000 by 17 equals 29,412 tons of recycled paper that would be saved. Multiplying 29,412 by 3.3 equals 97,060 cubic yards of landfill that would be saved.
  - Example #2: In a year, a family of four would produce (2.7 lbs.)(4 people)(365 days) = 3,942 lbs. of garbage. Dividing 3,942 by 50 equals 78.84 cubic feet of garbage. Dividing 78.84 by 27 equals 2.92 cubic yards of garbage.

Follow-up/extension

- Using the given information, have the students solve the following problem: Each year the United States generates about 450 million cubic yards of solid waste. Mrs. Robinson's classroom is 42 ft. long by 30 ft. wide by 12 ft. high. How many rooms of this size would be needed to hold the garbage generated by the United States in one year?

Waste Paper Recording Sheet

1. For every ton of paper that is recycled, about 17 trees and 3.3 cubic yards of landfill space are saved. If 500,000 trees are used each week to produce the Sunday newspapers, how much landfill would be saved if 100% of all newspapers used recycled paper one Sunday?

2. In the United States, an average of 2.7 pounds of garbage per person is delivered to available landfills each day. One cubic foot of compressed garbage weighs about 50 pounds.
   a. How many cubic yards of landfill space are used by a family of four in one year?

   b. How many cubic yards of landfill space are used by a class of 25 students in one year?

Sample released test items

The Vasquez family drove 165 miles in 3 hours. At this same rate, how many miles could they travel in 8 hours?

A 61.875
B 423.3
C 440
D 520

A taxi company based its fares on the following chart.

Miles	0.2	0.3	0.4	0.5
Fare	$0.60	$0.90	$1.20	$1.50

If the pattern continues, what would be the fare for a trip of 8.3 miles?

F $2.49
G $24.90
H $49.80
J $249.00

Eric is twice as old as his brother Lucas. If 4 is subtracted from Eric's age and 4 is added to Lucas's age, their ages will be equal. What are the boys' ages now?

F 12 and 6
G 14 and 7
H 16 and 8
J 18 and 9

An object in the current of the Gulf Stream can move 11 miles in 2 hours. At this rate, about how many miles could the object in the Gulf Stream move in 5 hours?

A 55

B $27\frac{1}{2}$

C 10

D $5\frac{1}{2}$

A factory that makes computer monitors produced 247 monitors during 5 working days. Which is *closest* to the number of working days that should be allowed to fill an order for 1,000 monitors?

F 15
G 20
H 30
J 40

A rock that weighed 1.2 pounds on the moon weighed 7.06 pounds on Earth. About how much would an astronaut who weighs 174 pounds on Earth weigh on the moon?

A 14.5 lbs
B 24.65 lbs
C 29.58 lbs
D 1,023.53 lbs

Roxanne's car used 4.8 gallons of gasoline to drive 124 miles. If Roxanne has 180 more miles to go, which is *closest* to the additional number of gallons of gasoline the car will use to complete the trip?

F 2.5

G 7.0

H 7.3

J 14.1

The sonar system of a submarine receives an echo back from a ship 5,000 yards away after 6.1 seconds. It picks up an echo from a second ship after 8.4 seconds. Which proportion could be used to find the distance to the second ship?

F $\frac{6.1}{5000} = \frac{8.4}{x}$

G $\frac{6.1}{8.4} = \frac{x}{5000}$

H $\frac{8.4 - 6.1}{8.4} = \frac{x}{5000}$

J $\frac{2.3}{5000} = \frac{6.1}{x}$

There are 48 newborn girls in a hospital nursery. For every 3 girls there are 2 boys. How many newborn boys are in the nursery?

F 72

G 48

H 32

J 24

Elizabeth drove 432 miles on the second day of a trip, which was 17 miles more than five times as far as she drove on the first day. How many miles did she drive on the first day?

A 61

B 83

C 84

D 90

Martha's grandfather at 63 years of age is 6 years more than 3 times as old as Martha. What is Martha's age?

F 17

G 19

H 21

J 23

Angie mows lawns in her neighborhood to make money. She charges $25 per lawn and buys a new mower for $200. If x is the number of lawns, and p is her profit, which of the following would you use to find Angie's profit?

A $25x - 200 = p$

B $200 = 25x + p$

C $25x = p - 200$

D $25x + 200 = p$

The table shows some elements of a function.

n	?
1	1
2	5
3	9
4	13

What is the missing rule in this table?

F $2n - 1$

G $2n + 1$

H $3n$

J $4n - 3$

The table shows some elements of a function.

n	?
1	$-\frac{1}{2}$
2	$\frac{1}{2}$
3	$\frac{3}{2}$
4	$\frac{5}{2}$

What is the missing rule in this table?

F $\dfrac{n + 1}{2}$

G $\dfrac{2n - 3}{2}$

H $\dfrac{n}{2}$

J $\dfrac{-n}{2}$

The table shows some elements of a function.

n	1	2	3	4
?	$\frac{3}{2}$	$\frac{4}{2}$	$\frac{5}{2}$	$\frac{6}{2}$

What is the missing rule in this table?

A $2n$

B $\dfrac{n + 2}{2}$

C $\dfrac{2n + 1}{2}$

D $3n$

x	-1	1	2	3
y	0	4	6	8

Which is true for all pairs of values in the table?

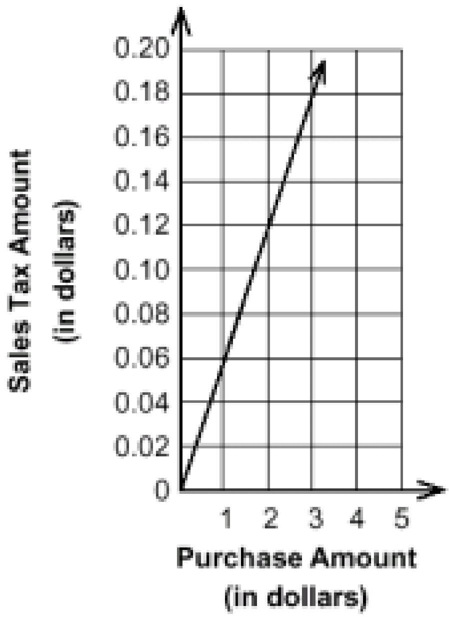
A $y = 2x - 2$

B $y = 2x + 2$

C $y = 4x$

D $y = x + 3$

The graph shows the amount of sales tax charged for purchases.



Which table best represents the information in this graph?

F

Purchase Amount	Sales Tax Amount
$1.00	$0.06
$2.00	$0.12
$3.00	$0.18
$4.00	$0.24

G

Purchase Amount	Sales Tax Amount
$1.00	$0.03
$2.00	$0.06
$3.00	$9.00
$4.00	$2.00

H

Purchase Amount	Sales Tax Amount
$1.00	$1.00
$2.00	$2.00
$3.00	$3.00
$4.00	$4.00

J

Purchase Amount	Sales Tax Amount
$1.00	$2.00
$2.00	$4.00
$3.00	$6.00
$4.00	$7.00

The table shows the relationship between d, the number of days a library book is overdue, and f, the amount of the fine.

d	1	2	3	4	5
f	$0.05	$0.10	$0.15	$0.20	$0.25

Which of the following describes the relationship?

F $f = 0.05d$

G $f = d + 0.05$

H $f = 2d + 0.05$

J $f = (d + 2)0.05$

Which table contains only values that satisfy $y = 3x - 5$?

A

x	y
2	1
0	$^{-}2$
2	$^{-}11$

B

x	y
1	2
3	4
5	10

C

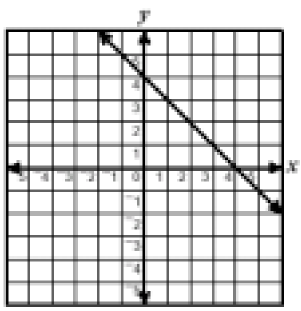
x	y
$^{-}3$	$^{-}14$
0	$^{-}5$
3	4

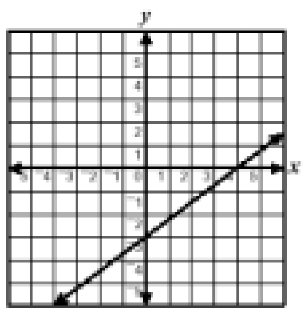
D

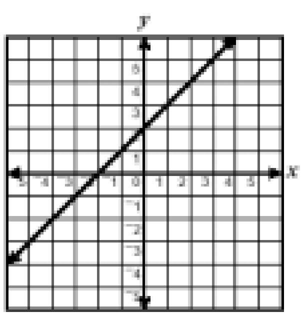
x	y
$^{-}5$	$^{-}15$
0	$^{-}5$
5	0

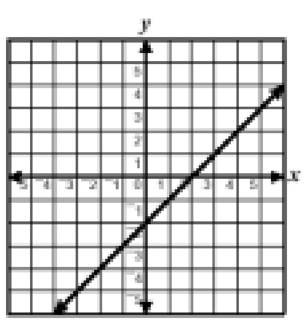
x	y
3	1
0	$^{-}2$
$^{-}3$	$^{-}5$

Which graph shows a line that contains the points in the table of ordered pairs?

A


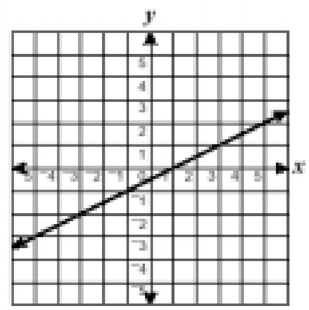
B


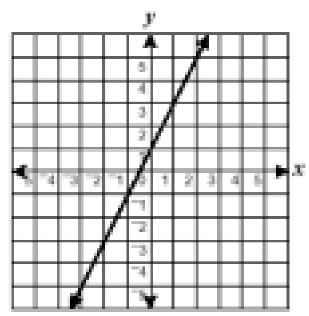
C


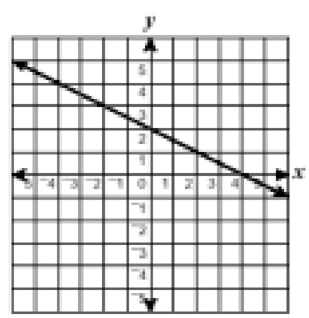
D


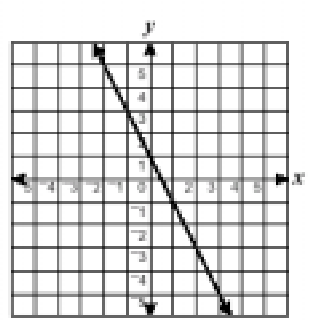
x	y
2	5
0	1
$^{-}2$	$^{-}3$

Which graph shows a line that contains the points in the table of ordered pairs?

A


B


C


D


Which table shows ordered pairs that satisfy the function $y = 3x + 1$?

F

x	y
0	1
2	7
4	13

G

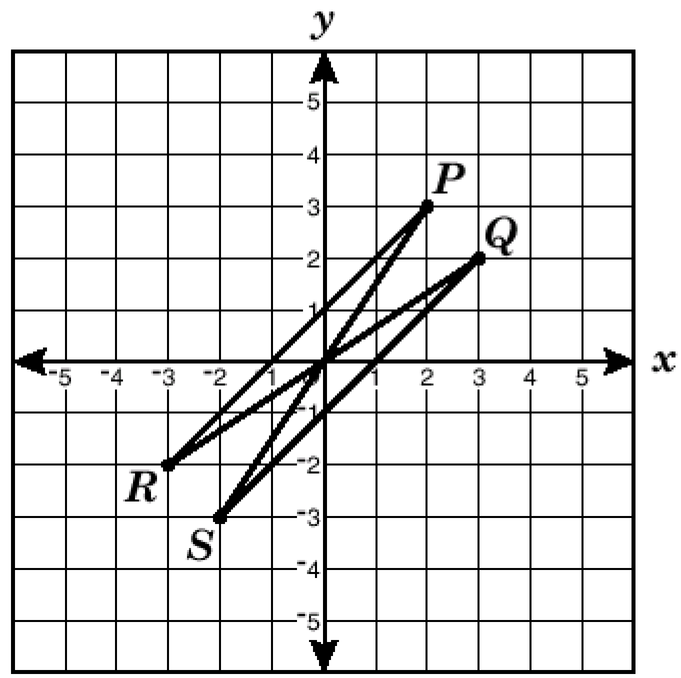
x	y
−2	1
0	7
2	13

H

x	y
−2	−5
0	0
2	7

J

x	y
0	4
2	7
4	13



Which line segment connects (2, 3) and (−3, −2)?

A $\overline{PQ}$

B $\overline{PR}$

C $\overline{QS}$

D $\overline{RS}$

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n	1	2	3	4
?	$\frac{3}{2}$	$\frac{4}{2}$	$\frac{5}{2}$	$\frac{6}{2}$

What is the missing rule in this table?

A $2n$

B $\frac{n + 2}{2}$

C $\frac{2n + 1}{2}$

D $3n$

The table shows some elements of a function.

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1	$^-\frac{1}{2}$
2	$\frac{1}{2}$
3	$\frac{3}{2}$
4	$\frac{5}{2}$

What is the missing rule in this table?

F $\frac{n + 1}{2}$

G $\frac{2n - 3}{2}$

H $\frac{n}{2}$

J $\frac{-n}{2}$

x	$^-1$	1	2	3
y	0	4	6	8

Which is true for all pairs of values in the table?

A $y = 2x - 2$
B $y = 2x + 2$
C $y = 4x$
D $y = x + 3$

The table shows the relationship between *d*, the number of days a library book is overdue, and *f*, the amount of the fine.

d	1	2	3	4	5
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Which of the following describes the relationship?

F $f = 0.05d$

G $f = d + 0.05$

H $f = 2d + 0.05$

J $f = (d + 2)0.05$

Which table shows ordered pairs that satisfy the function $y = 3x + 1$?

F

x	*y*
0	1
2	7
4	13

G

x	*y*
-2	1
0	7
2	13

H

x	*y*
-2	-5
0	0
2	7

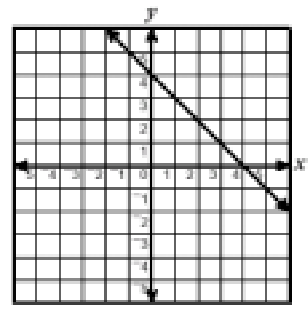
J

x	*y*
0	4
2	7
4	13

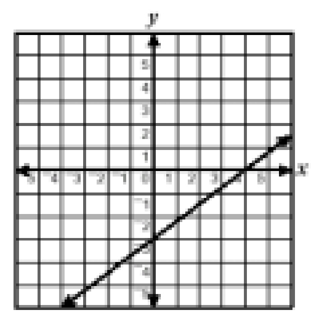
x	*y*
3	1
0	-2
-3	-5

Which graph shows a line that contains the points in the table of ordered pairs?

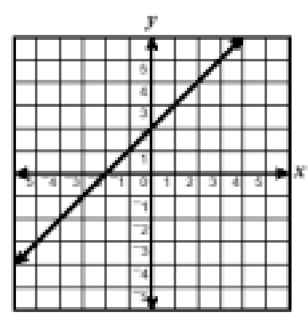
A



B



C



D